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Riemannian Geometry

sheet 09

Exercise 1. Show that $S^2 \times S^2$ does not admit a metric of constant sectional curvature.

Exercise 2. Let G be a Lie group such that $[X, Y] \neq 0$ for all linearly independent elements in its Lie algebra. Assuming that G carries a bi-invariant metric, show that G must be compact.

Exercise 3. For nonzero integers p, q define a group $BS_{p,q} = \langle \alpha, \beta \mid \beta\alpha^p\beta^{-1} = \alpha^q \rangle$. Show that $BS_{p,q}$ cannot be a subgroup of the fundamental group of any connected compact Riemannian manifold with strictly negative sectional curvature.

Exercise 4. Consider the product $M \times N$ of two connected compact manifolds which are not simply connected ($\pi_1(M) \neq 0 \neq \pi_1(N)$). Show that $M \times N$ cannot admit a metric of strictly negative sectional curvature.