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# Riemannian Geometry

## sheet 03

**Exercise 1.** Show that $(\mathbb{H}, g^{hyp})$ from Ex. 3, sheet 02, has constant sectional curvature $-1$.

**Exercise 2.** Recall that a Riemannian metric $g$ is called Einstein with Einstein constant $\lambda$ if $Ric = \lambda g$. Let $(M, g^N)$ and $(N, g^M)$ be Einstein manifolds with Einstein constants $\lambda_M$, $\lambda_N$. Show that the product metric on $M \times N$ is Einstein if and only if $\lambda_M = \lambda_N$.

**Exercise 3.** Let $(M, g)$ be a 4-dimensional Riemannian manifold. Show that $g$ is Einstein if and only if for every point $p \in M$ and every 2-plane $\sigma \subseteq T_pM$ with orthogonal complement $\sigma^\perp$ one has $K(\sigma) = K(\sigma^\perp)$. **Hint:** Consider orthogonal bases for $\sigma$ and $\sigma^\perp$

**Exercise 4.** Let $(M, g)$ be a 4-dimensional Riemannian manifold.

1. Assume that for every point $p \in M$ and every 2-plane $\sigma \subseteq T_pM$ with orthogonal complement $\sigma^\perp$ one has $K(\sigma) = -K(\sigma^\perp)$. Show that the scalar curvature of $g$ vanishes identically.
2. Show that the product metric on $S^2 \times \mathbb{H}^2$ satisfies the assumption of the previous point.