MATHEMATISCHES INSTITUT
DER UNIVERSITÄT MÜNCHEN
Prof. Dr. S. Bachmann / Dr. P. Soneji

WS 2014/15
– Blatt 14 –
21.1.2015

PDG I
(Tutorium)

Tutorial 14
(Radial maps and Sobolev Spaces)

Suppose $1 \leq p < \infty$, $R > 0$, and let B_R denote the ball $B(0, R)$ in $\mathbb{R}^n$, where $n > 1$. Furthermore, let $f \in C^\infty(0, \infty)$ and define

$$u(x) := f(|x|)\,, \quad x \in B_R\,.$$

It is easy to show, using polar coordinates, that $u \in L^p(B_R)$ if and only if

$$\int_0^R r^{n-1}|f(r)|^p \,\mathrm{d}r < \infty\,.$$

Now suppose further that

$$\lim_{r\searrow 0} \left(r^{n-1}|f(r)|\right) = 0\,.$$

Then we shall show that $u \in W^{1,p}(B_R)$ if and only if $u \in L^p(B_R)$ and

$$\int_0^R r^{n-1}|f'(r)|^p \,\mathrm{d}r < \infty\,.$$

We may now apply this to a number of different f to obtain several examples of radial Sobolev maps.

(a) For $u(x) = |x|^{-s}$, where $s > 0$, $u \in W^{1,p}(B_R)$ if and only if $1 \leq p < \frac{n}{s+1}$.

(b) For $u(x) = \log(|x|)$, $u \in W^{1,p}(B_R)$ if and only if $1 \leq p < n$.

(c) For $u(x) = \frac{x}{|x|}$ $(\in \mathbb{R}^n)$, $R = 1$, $u \in W^{1,p}(B_1; \mathbb{R}^n)$ if and only if $1 \leq p < n$.