MATHEMATISCHES INSTITUT
DER UNIVERSITÄT MÜNCHEN
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PDG I
(Tutorium)

Tutorial 13

(Weak derivatives and Sobolev Spaces)

In this tutorial we covered the following topics:

- Approximation of weak derivatives with smooth functions, using mollifiers. In particular, if $u \in L^1_{\text{loc}}(\Omega)$ has a weak derivative $\frac{\partial u}{\partial x_i}$, $E \subset\subset \Omega$, and ϱ_ϵ is a mollifier (where $0 < \epsilon < \text{dist}(E, \partial\Omega)$, then for $x \in E$,

$$\frac{\partial}{\partial x_i}(\varrho_\epsilon * u)(x) = \left(\varrho_\epsilon * \frac{\partial u}{\partial x_i}\right)(x)\,.$$

- If $u \in L^1_{\text{loc}}(\Omega)$ has all weak derivatives equal to zero, then it is (almost everywhere) constant.

- A discussion of some properties of the Sobolev spaces $W^{1,p}(\Omega)$: e.g. density of smooth functions in $W^{1,p}(\Omega)$ for $1 \leq p < \infty$; if Ω is bounded and $\partial\Omega$ is Lipschitz, then $u \in W^{1,\infty}(\Omega)$ if and only if it is Lipschitz continuous.