Mathematisches Institut der LMU
Helmut Schwichtenberg

Winter semester 2025/26
hwk09

Homework for the lecture course „Mathematical Logic“

Problem 33. (4 points). The Gauß-sum $\mathrm{Gs}(n)$ is the sum of the first n natural numbers. Prove informally

(a) $\mathrm{Gs}(n) = \frac{n(n+1)}{2}$.

(b) $\mathrm{Gs}(n+1) + \mathrm{Gs}(n) = (n+1)^2$.

Hint. Recall the Gauß code of pairs of natural numbers

$$\begin{array}{cccccc}
15 & \ldots & & & & \\
10 & 16 & \ldots & & & \\
6 & 11 & \ldots & & & \\
3 & 7 & 12 & \ldots & & \\
1 & 4 & 8 & 13 & \ldots & \\
0 & 2 & 5 & 9 & 14 & \ldots
\end{array}$$

Problem 34. (4 points). Prove that the following functions are elementary

(a) $\lfloor \frac{n}{m} \rfloor$.

(b) $n \bmod m$.

Problem 35. (4 points). Prove that the class $\mathcal{E}$ is closed under limited course-of-values recursion. The means: if g, h, k are given functions in $\mathcal{E}$ and f is defined by the scheme

$$\begin{aligned}
f(\vec{m}, 0) &= g(\vec{m}), \\
f(\vec{m}, n+1) &= h(n, \langle f(\vec{m}, 0), \ldots f(\vec{m}, n)\rangle, \vec{m}), \\
f(\vec{m}, n) &\le k(\vec{m}, n),
\end{aligned}$$

then f is in $\mathcal{E}$.

Problem 36. (4 points). Formalize the proofs from Problem 33 in Minlog (see `ueb09.scm`).

Due. Wednesday, 17. December 2025, 8:00.