Mathematisches Institut der LMU
Helmut Schwichtenberg

Winter semester 2025/26
hwk03

Homework for the lecture course „Mathematical Logic“

Problem 9. (4 points). Give derivations for
(a) $(\neg\neg A \to A) \to (\neg\neg B \to B) \to \neg\neg(A \wedge B) \to A \wedge B$
(b) $\forall_{x,y}(Pxy \to Qxy) \to \forall_x\exists_y(Qxy \to Rxy) \to \forall_x\exists_y(Pxy \to Rxy)$

Problem 10. (4 points). Prove by induction on formulas that for all formulas A one can derive

$$\vdash_i \bot \to A.$$

Use here the rules for the connectives $\to$, $\forall$, $\vee$, $\wedge$, $\exists$. In the existential case we must know that the domain of objects is not empty, for instance by assuming $\exists_x(A \to A)$ with a nullary relation symbol A.

Problem 11. (4 points). Consider disjunction, introduced by the axioms

$$\vee_0^+ \colon A \to A \vee B,$$
$$\vee_1^+ \colon B \to A \vee B,$$
$$\vee^- \colon A \vee B \to (A \to C) \to (B \to C) \to C,$$

or by the rules

$$\frac{\begin{array}{c}\mid M\\ A\end{array}}{A \vee B}\vee_0^+ \qquad \frac{\begin{array}{c}\mid M\\ B\end{array}}{A \vee B}\vee_1^+ \qquad \frac{\begin{array}{ccc}& [u\colon A] & [v\colon B]\\ \mid M & \mid N & \mid K\\ A \vee B & C & C\end{array}}{C}\vee^- u, v$$

(a) Show that the axioms are derivable from the rules.
(b) Let ax-derivations be those with $\vee$-axioms but without $\vee$-rules. Show
  (i) For every ax-derivation $M \colon A$ there is an ax-derivation of $A \vee B$.
  (ii) For every ax-derivation $N \colon B$ there is an ax-derivation of $A \vee B$.
  (iii) For ax-derivations $M \colon A \vee B$, $N \colon C$ and $K \colon C$ with open assumptions $u \colon A$ in N and $v \colon B$ in K there is an ax-derivation $L \colon C$ with no free occurrences of u, v.
(c) Infer that adding the $\vee$-rules or the $\vee$-axiom makes the same formulas derivable.

Problem 12. (4 points). Formalize the derivations from Problems 9 and 10 in Minlog (see `ueb03.scm`).

Due. Wednesday, 5. November 2025, 8:00