Mathematisches Institut der LMU
Helmut Schwichtenberg

Winter semester 2025/26
hwk02

Homework for the lecture course „Mathematical Logic“

Problem 5. (4 points). Substitution in terms is recursively defined by

$$\begin{aligned} x[x/s] &:= s, \\ y[x/s] &:= y \quad \text{if } y \text{ is different from } x, \\ c[x/s] &:= c, \\ (ft_1 \dots t_n)[x/s] &:= ft_1[x/s] \dots t_n[x/s]. \end{aligned}$$

Define recursively substitution $A[x/t]$ for formulas. Note that occasionally vaiables need to be renamed. Example:

$$(\forall_x Pxy)[y/fx] = \forall_z Pz(fx).$$

Problem 6. (4 points). Give derivations for

$$(A \to B) \to \neg B \to \neg A,$$
$$\neg(A \to B) \to \neg B,$$
$$\neg\neg(A \to B) \to \neg\neg A \to \neg\neg B,$$
$$(\bot \to B) \to (\neg\neg A \to \neg\neg B) \to \neg\neg(A \to B),$$
$$\neg\neg\forall_x A \to \forall_x \neg\neg A.$$

Problem 7. (4 points). Give derivations for

$$(A \lor B \to C) \leftrightarrow (A \to C) \land (B \to C),$$
$$(A \to B) \lor (A \to C) \to A \to B \lor C.$$

In the first formula derivations for both directions are needed.

Problem 8. (4 points). Formalise the three derivations from Problem 7 in Minlog (see `ueb02.scm`).

Due. Wednesday, 29. October 2025, 8:00