G1 in 5 min

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Reference


Brendan Cordy
Diagonalization & The Fixed Point Lemma
http://qubd.github.io/files/FixedPointLemma.pdf,


Yong Cheng
Current research on Gödel's incompleteness theorems
Bulletin of Symbolic Logic, 2021, 113-167

Self-Reference

-Can you write a statement which is true iff Alice is reading it?

- *Yes! 'Alice is reading this statement.' Because* ***this statement is true*** *iff* ***Alice is reading this statement*** *iff* ***Alice is reading it!***

Self-Reference

-Can you write a statement which is true iff Alice is reading it?

◦ *Yes! 'Alice is reading this statement.' Because* ***this statement is true*** *iff* ***Alice is reading this statement*** *iff* ***Alice is reading it!***

-What is 'this statement' ?

◦ *'Alice is reading this statement.'*

-What is 'this statement' exactly?!

◦ *'Alice is reading this statement.'*

-.....

Congratulations! Now you know what is 'Self-reference' (or 'Self-contain').

-I don't like 'this'. Please give an answer without use of the annoying 'this'.

∘*emm, how about*

Alice is reading the statement 'Alice is reading the statement'

-I don't like 'this'. Please give an answer without use of the annoying 'this'.

∘*emm, how about*

Alice is reading the statement 'Alice is reading the statement'

-Kinda not working.
-Because this statement is true
if and only if **Alice** is reading the statement '**Alice** is reading the statement',
which is not the same as **Alice** is reading "**Alice** is reading the statement '**Alice** is reading the statement".

Substition

Let $r(x)$ be 'My favorite sentence is x', where x is called a **variable**.
If c means *Good morning*, then $r(c)$ means

My favorite sentence is 'Good morning'.

If b means *My favorite sentence is* x, then $r(b)$ means

My favorite sentence is 'My favorite sentence is x*'.*

Notice: the x here is not a **variable**!

Diagonalization

So for any formula $s(x)$ with one variable, we call $s(\text{`}s(x)\text{'})$ its diagonalization.
Then consider the sentence

Alice is reading the diagonalization of 'Alice is reading the diagonalization of x'

Bingo!

What did Gödel do?

A statement which is true iff Alice is reading it?

Fixed Point

What did Gödel do?

A statement which is true iff Alice is reading it?

Fixed Point

Then you can similarly get a statement that

The statement is true iff it has no proof (in PA.)

Get $G1$!
Non-trivial and technical parts :

- Code/Arithmetization
- The Diagonalization function is recursive.
- The notion of 'proof' and related concepts in $G1$ and $G2$ are then expressed ('represented') via arithmetization.