Hyperintensional Partial Typed λ-calculus

Proof & Computation in Herrsching

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Tichy's Transparent Intensional Logic

TIL Foundations

- Shift from set-theoretical semantics to *procedural semantics*.
- Shift from mathematical functions (set-theoretical mappings) to algorithmically structured procedures.
- Shift from Church's STT to Tichy's modification of Russell's RTT.

Motivation behind partiality

- What is the meaning (extension) of the linguistic expression: *Two divided by zero.*

$$[\div \mathbf{2}\ \mathbf{0}] \to \bot \tag{1}$$

$$[* [\div \mathbf{2}\ \mathbf{0}]\ \mathbf{3}] \to \bot \tag{2}$$

$$[= [* [\div \mathbf{2}\ \mathbf{0}]\ \mathbf{3}]\ \mathbf{5}] \to \bot \tag{3}$$

Tichy's Transparent Intensional Logic

Motivation behind partiality

- Should this *improperness* be propagated strictly from bottom up in any case?

$$\lambda x[\div\ \mathbf{2}\ \mathbf{0}] \overset{?}{\to} \bot \quad \text{not, but} \quad \lambda x[\div\ \mathbf{2}\ \mathbf{0}] \to f,$$

where f is a degenerate function, which is not defined for any argument.

- This is called *intensional lift* of context in TIL
- Key point – distinguish between constructions and values produced by them.

Tichy's Transparent Intensional Logic

Motivation behind Hyperintensionality

- What is the meaning (extension) of the linguistic expression: *Two divided by zero is not defined.*

$$[\mathbf{Improp}\ [\div\ \mathbf{2}\ \mathbf{0}]] \overset{?}{\rightarrow} \bot \quad \text{not, but} \quad [\mathbf{Improp}\ {}^{0}[\div\ \mathbf{2}\ \mathbf{0}]] \rightarrow True$$

- This is called *hyperintensional lift* in TIL.
- Key point – distinguish between procedures and the values produced by them.

Questions