Substructural Logics with Modal Operators

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About the research project

We aim to investigate cut-free dual context sequent calculus for substructural modal logics.

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4. This concept was first introduced by Jean-Yves Girard in 1993 [4].

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2. Contraction

Example

See modal logic K by Pfenning (Slides)

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