HOLMS: HOL LIGHT LIBRARY FOR MODAL SYSTEMS

Antonella Bilotta — Scuola Normale Superiore

Marco Maggesi — Università degli Studi Firenze

Cosimo Perini Brogi — IMT Scuola Alti Studi Lucca

Proof and Computation 2025 — September, 2025

AGENDA

1. What is HOLMS?

2. Mechanising Modal Logics

3. HOLMS Architecture

WHAT IS HOLMS?

HOLMS

stands for

HOL Light Library for Modal Systems

Is a framework for modal reasoning within the proof assistant HOL Light.

Modal Logics

HOL Light

Mechanisation of Modal Logics

WHAT IS HOLMS?

HOLMS

stands for

HOL Light Library for Modal Systems

Is a framework for modal reasoning within the proof assistant HOL Light.

Modal Logics

HOL Light

Mechanisation of Modal Logics

WHAT IS HOLMS?

HOLMS

stands for

HOL Light Library for Modal Systems

Is a framework for modal reasoning within the proof assistant HOL Light.

Modal Logics

HOL Light

Mechanisation of Modal Logics

WHAT IS HOLMS?

HOLMS

stands for

HOL Light Library for Modal Systems

Is a framework for modal reasoning within the proof assistant HOL Light.

Modal Logics

HOL Light

Mechanisation of Modal Logics

To mechanise a modal system means to develop **formal or computational tools** to **represent**, **analyse**, and **manipulate** it.

ML Metalanguage (OCaml)

`HOLMS_RULE` $`\forall A.\ \vdash_{K4} \Box A \longrightarrow \Box\Box A'$

Terms and Theorems Language (HOL Light)

$\vdash \forall A.(\vdash_{K4} A \implies \vdash_{K4} \Box A)$

$\vdash \forall A.(\vdash_{K4} \Box A \longrightarrow \Box\Box A)$

Object Theory Language $\mathcal{L}_{mod}$ (K4)

$\vdash_{K4} \Box A \longrightarrow \Box\Box A$

The problem of mechanising modal systems has given rise to a rich body of scientific work, including:

- Theoretical contributions [18] [20, 1]
- Implementations within proof assistants:
  - **Prolog** [5, 6] [10, 11, 12, 13]; - **Coq/Rocq** [7] [14];
  - **Isabelle/HOL** [8, 9] [3, 2]; - **HOL Light** [15] [16, 17]

Despite a high degree of modularity, existing approaches lack tools for **assessing portability** across **modal logics** and **proof assistants**, as well as metrics for evaluating compositional design.

⟶ HOLMS was developed as a **modular** extension of the library for GL:

- **Portable** across **modal logics**;
- **Portable** between various **theorem provers**.

HOLMS

stands for

HOL Light Library for Modal Systems

Is a framework for modal reasoning within the proof assistant HOL Light.

HOLMS

↓

$\vdash_{\mathbb{S}} \quad A\ ?$

↓

Automated Proof Search

Yes → **Certified Theorem for** $\mathbb{K}$, $\mathbb{T}$, $\mathbb{K}4$, $\mathbb{S}4$, $\mathbb{B}$, $\mathbb{S}5$, and $\mathbb{GL}$

No → **Certified Countermodel Construction for** $\mathbb{K}$, $\mathbb{T}$, $\mathbb{K}4$, $\mathbb{S}4$, $\mathbb{B}$, $\mathbb{S}5$, and $\mathbb{GL}$

Implementation methodology behind HOLMS:

Axiomatic calculus ⋮⋮⋮⋮⋮⋮⋮ Frame class characterising modal schemas

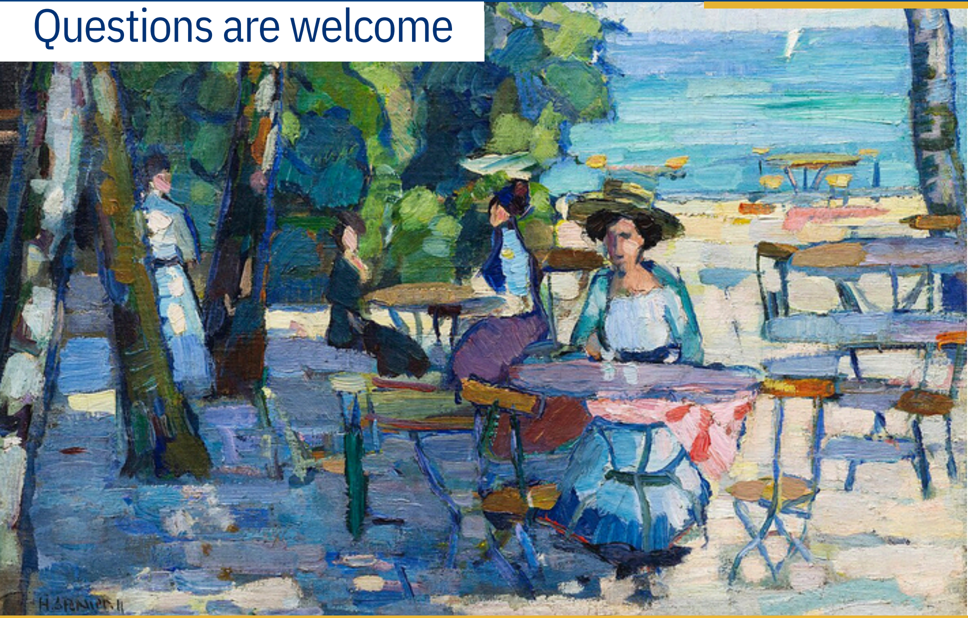
Decision procedure .... Labelled sequent calculus

.... shallow embedding via a HOL Light rule for proof-search

:::: formalised adequacy theorems

THANK YOU FOR LISTENING!

Questions are welcome



THE GOAL OF PARAMETRICTY

The **ultimate goal** of the project is to develop a **general mechanism for automated modal reasoning**.

▷ The **focus** of HOLMS implementation methodology is to make the code as ***parametric*** as possible.

- To precisely **measure** and **enhance code modularity**, we adopted a precise coding discipline:
  (a) **Parametric Polymorphic**: code **fully independent** of specific parameters instantiations;
  (b) **Ad-hoc Polymorphic**: code whose components are **tailored** to the modal logic under consideration.

MEASURING MODULARITY

This version of HOLMS allows us to measure the parametricity of the code, and to make the implementation schema more informative:

Axiomatic calculus ··· Frame class characterising modal schemas

Decision procedure ··· Labelled sequent calculus

.... shallow embedding via a HOL Light rule for proof-search

=== formalised adequacy theorems

Parametric   Ad-hoc Polymorphic

Portability across modal logics

Syntax		Parametric
Semantics		Parametric
Correspondence Theory	Concepts	Parametric
	Lemmata	Ad-hoc Polimorphic
Soundness		Parametric
Completeness	"Standard" Model	Parametric
	Truth Lemma	Parametric
	Counteromodel Lemma	Parametric
	"Standard" Relation	Ad-hoc Polimorphic
	Identification of the "Standard" Model	Ad-hoc Polimorphic
Shallow Embedding		Ad-hoc Polimorphic
Decision		Ad-hoc Polimorphic

- We maximised the parametrization of the **adequacy theorems**, producing from [4] a uniform proof that can be **easily extended to various modal logics**. To reach this result, we relied on **correspondence theory**.
- The use of modular labelled sequent calculi for modal systems [19] allow us to provide **shallow embeddings** and **decision procedures for a wide range of modal logics**.

BIBLIOGRAPHY I

[1] Christoph Benzmüller.
Faithful logic embeddings in HOL - A recipe to have it all: deep and shallow, automated and interactive, heavy and light, proofs and counterexamples, meta and object level.
CoRR, abs/2502.19311, 2025.

[2] Christoph Benzmüller and Bruno Woltzenlogel Paleo.
Higher-order modal logics: Automation and applications.
In Wolfgang Faber and Adrian Paschke, editors, *Reasoning Web. Web Logic Rules - 11th International Summer School 2015, Berlin, Germany, July 31 - August 4, 2015, Tutorial Lectures*, volume 9203 of *Lecture Notes in Computer Science*, pages 32–74. Springer, 2015.

[3] Christoph Benzmüller, Maximilian Claus, and Nik Sultana.
Systematic verification of the modal logic cube in isabelle/hol.
In Cezary Kaliszyk and Andrei Paskevich, editors, *Proceedings Fourth Workshop on Proof eXchange for Theorem Proving, PxTP 2015, Berlin, Germany, August 2-3, 2015*, volume 186 of *EPTCS*, pages 27–41, 2015.

BIBLIOGRAPHY II

[4] George Boolos.
The logic of provability.
Cambridge University Press, 1995.

[5] Tiziano Dalmonte, Sara Negri, Nicola Olivetti, and Gian Luca Pozzato.
Theorem Proving for Non-normal Modal Logics.
In *OVERLAY 2020*, Udine, Italy, September 2021.

[6] Tiziano Dalmonte, Sara Negri, Nicola Olivetti, Gian Luca Pozzato, and Cyril Terrioux.
PRONOM: A theorem prover for nonnormal modal logics.
Available at `http://193.51.60.97:8000/pronom/`.

[7] Christian Doczkal and Gert Smolka.
Completeness and decidability results for CTL in constructive type theory.
Journal of Automated Reasoning, 56(3):343–365, 2016.

BIBLIOGRAPHY III

[8] Asta Halkjær From.
Formalized soundness and completeness of epistemic logic.
In Alexandra Silva, Renata Wassermann, and Ruy J. G. B. de Queiroz, editors, *Logic, Language, Information, and Computation - 27th International Workshop, WoLLIC 2021, Virtual Event, October 5-8, 2021, Proceedings*, volume 13038 of *Lecture Notes in Computer Science*, pages 1–15. Springer, 2021.

[9] Asta Halkjær From.
An isabelle/hol framework for synthetic completeness proofs.
In *Proceedings of the 14th ACM SIGPLAN International Conference on Certified Programs and Proofs*, CPP '25, page 171–186, New York, NY, USA, 2025. Association for Computing Machinery.

BIBLIOGRAPHY IV

[10] Marianna Girlando, Bjoern Lellmann, Nicola Olivetti, Stefano Pesce, and Gian Luca Pozzato.
Theorem proving for Lewis Logics of Counterfactual Reasoning.
In *CILC 2020 - 35th Edition of the Italian Conference on Computational Logic*, Rende / Virtual, Italy, October 2020.

[11] Marianna Girlando, Björn Lellmann, Nicola Olivetti, Gian Luca Pozzato, and Quentin Vitalis.
Vinte: An implementation of internal calculi for lewis' logics of counterfactual reasoning.
In Renate A. Schmidt and Cláudia Nalon, editors, *Automated Reasoning with Analytic Tableaux and Related Methods*, pages 149–159, Cham, 2017. Springer International Publishing.

[12] Marianna Girlando and Marianela Morales.
MOILab: towards a labelled theorem prover for intuitionistic modal logics.
working paper or preprint, December 2020.

BIBLIOGRAPHY V

[13] Marianna Girlando and Lutz Straßburger.
Moin: A nested sequent theorem prover for intuitionistic modal logics (system description).
In Nicolas Peltier and Viorica Sofronie-Stokkermans, editors, *Automated Reasoning*, pages 398–407, Cham, 2020. Springer International Publishing.

[14] C Hagemeier.
Formalizing intuitionistic epistemic logic in Coq, 2021.

[15] John Harrison.
HOL Light tutorial.
Intel Corporation.
http://www.cl.cam.ac.uk/~jrh13/hol-light/tutorial.pdf, 2017.

BIBLIOGRAPHY VI

[16] Marco Maggesi and Cosimo Perini Brogi.
A formal proof of modal completeness for provability logic.
In Liron Cohen and Cezary Kaliszyk, editors, *12th International Conference on Interactive Theorem Proving (ITP 2021)*, volume 193 of *Leibniz International Proceedings in Informatics (LIPIcs)*, pages 26:1–26:18, Dagstuhl, Germany, 2021. Schloss Dagstuhl – Leibniz-Zentrum für Informatik.

[17] Marco Maggesi and Cosimo Perini Brogi.
Mechanising Gödel-Löb Provability Logic in HOL Light.
Journal of Automated Reasoning, 67(3):29, 2023.

[18] Cláudia Nalon, Ullrich Hustadt, Fabio Papacchini, and Clare Dixon.
Buy one get 14 free: Evaluating local reductions for modal logic.
In Brigitte Pientka and Cesare Tinelli, editors, *Automated Deduction – CADE 29*, pages 382–400, Cham, 2023. Springer Nature Switzerland.

[19] Sara Negri.
Proof analysis in modal logic.
Journal of Philosophical Logic, 34(5):507–544, 2005.

[20] Alexander Steen, Geoff Sutcliffe, and Christoph BenzmÜller.
Solving quantified modal logic problems by translation to classical logics.
Journal of Logic and Computation, page exaf006, 2025.