Characterizing the expressiveness of inconsistency-tolerant modal logics

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Logics of Formal Inconsistency

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- $\mathfrak{M}, w \nVdash \breve{\circ}\varphi$ iff $\mathfrak{M}, w \Vdash \varphi$ and $\mathfrak{M}, w \Vdash \smile\varphi$

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    - "Given a first order formula φ, is there a modal formula that its translation is φ?" Characterize the first order fragment corresponding to the modal logic as the "bisimulation-invariant" fragment
- Classical negation is not definable over the class of all Kripke Frames in $\mathcal{L}$
- Bisimulations results for a bigger class of languages than $\mathcal{L}$

Thank you!