

# BB(5) in 5 minutes

The bbchallenge Collaboration  
[bbchallenge.org](http://bbchallenge.org)



ERC No 772766, SFI 18/ERCS/5746

Does it halt?

	0	1
A	1RB	1LC
B	1RC	1RB
C	1RD	0LE
D	1LA	1LD
E	---	0LA

Step #0

0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---

Does it halt?

	0	1
A	1RB	1LC
B	1RC	1RB
C	1RD	0LE
D	1LA	1LD
E	---	0LA

Step #1

0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0
---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---

Does it halt?

	0	1
A	1RB	1LC
B	1RC	1RB
C	1RD	0LE
D	1LA	1LD
E	---	0LA

Step #2

0	0	0	0	0	0	0	0	1	1	0	0	0	0	0	0	0
---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---

Does it halt?

	0	1
A	1RB	1LC
B	1RC	1RB
C	1RD	0LE
D	1LA	1LD
E	---	0LA

Step #3

0	0	0	0	0	0	0	0	1	1	1	0	0	0	0	0	0
---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---

Does it halt?

	0	1
A	1RB	1LC
B	1RC	1RB
C	1RD	0LE
D	1LA	1LD
E	---	0LA

Step #4

0	0	0	0	0	0	0	0	1	1	1	1	0	0	0	0	0
---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---

Does it halt?

	0	1
A	1RB	1LC
B	1RC	1RB
C	1RD	0LE
D	1LA	1LD
E	---	0LA

Step #5

0	0	0	0	0	0	0	0	1	1	1	1	0	0	0	0	0
---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---

Does it halt?

	0	1
A	1RB	1LC
B	1RC	1RB
C	1RD	0LE
D	1LA	1LD
E	---	0LA

Step #6

0	0	0	0	0	0	0	0	1	0	1	1	0	0	0	0	0	0
---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---



Does it halt?

	0	1
A	1RB	1LC
B	1RC	1RB
C	1RD	0LE
D	1LA	1LD
E	---	0LA

Step #6

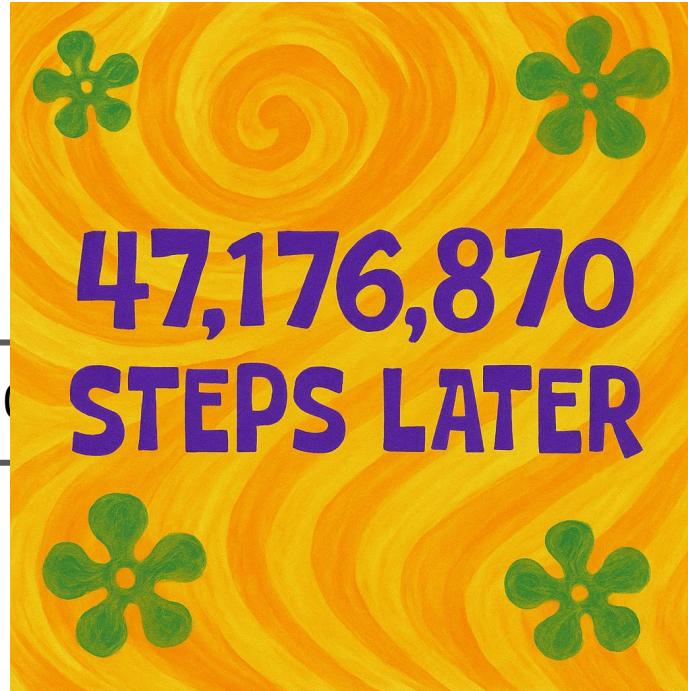
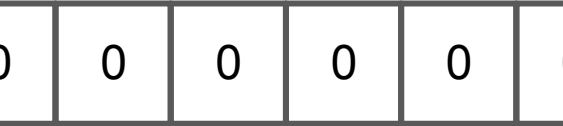
Will we ever read a 0 in state E ?

0	0	0	0	0	0	0	0	1	0	1	1	0	0	0	0	0
---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---

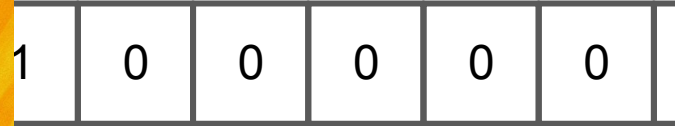
Does it halt?

	0	1
A	1RB	1LC
B	1RC	1RB
C	1RD	0LE
D	1LA	1LD
E	---	0LA

Step #6



er read a 0 in state E ?



Does it halt?  
Yes!

	0	1
A	1RB	1LC
B	1RC	1RB
C	1RD	0LE
D	1LA	1LD
E	---	0LA

Step #47,176,870

0	0	0	0	0	0	0	1	0	1	0	0	1	0	0	1	0	
---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	--

The machine has halted!!

Does it halt?

	0	1
A	1RB	1LC
B	1RC	1RB
C	1RD	0LE
D	1LA	1LD
E	---	0LA

Step #47,176,870

0	0	0	0	0	0	0	1	0	1	0	0	1	0	0	1	0	
---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	--

The machine has halted!!

Can another 5-state machine do better?

# The Busy Beaver function

$BB(n)$  = “Maximum number of steps done by a halting 2-symbol Turing machine with  $n$  states starting from all-0 memory tape”

T. Radó. On Non-computable Functions. *Bell System Technical Journal*, 41(3):877–884. 1962.

$BB(n)$  = “Maximum algorithmic bang for your buck”

UNCOMPUTABLE



Tibor Radó, 1895 - 1965

# The Busy Beaver function

$BB(n)$  = “Maximum number of steps done by a halting 2-symbol Turing machine with  $n$  states starting from all-0 memory tape”

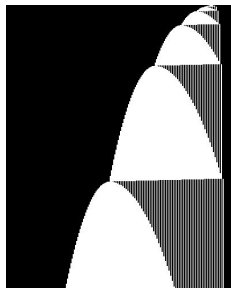
T. Radó. On Non-computable Functions. *Bell System Technical Journal*, 41(3):877–884. 1962.

Small busy beaver values:

- $BB(1) = 1$ ,  $BB(2) = 6$  [Radó, 1962]
- $BB(3) = 21$  [Radó and Lin, 1963]
- $BB(4) = 107$  [Brady, 1983]
- $BB(5) \geq 47,176,870$  [Marxen and Buntrock, 1989]

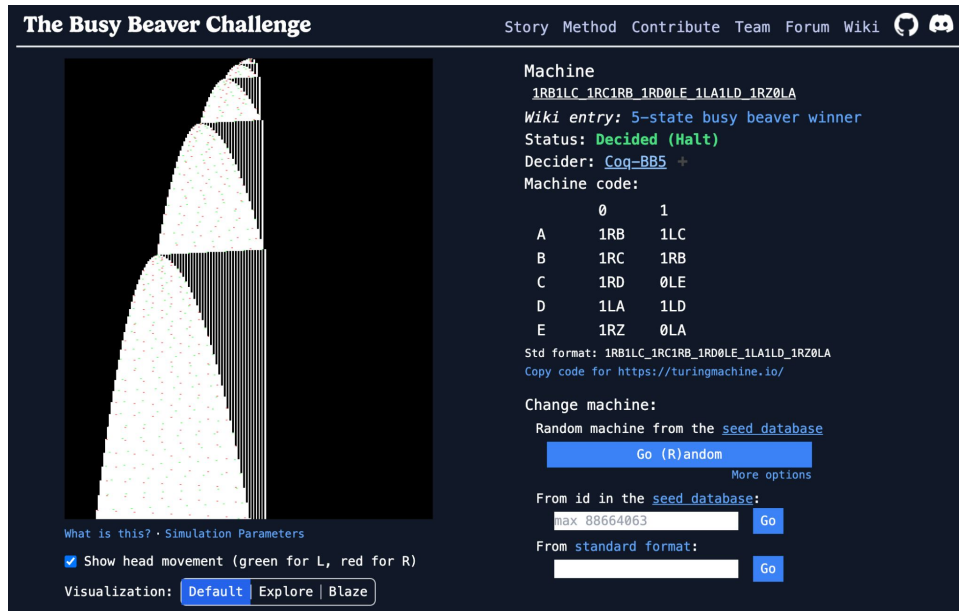
$BB(5) = 47,176,870?$

Conjectured yes [Aaronson, 2020]



# The Busy Beaver Challenge (bbchallenge)

Launched in 2022



The screenshot shows the 'The Busy Beaver Challenge' website. On the left, there's a visualization of a Turing machine's tape, showing a pattern of white and black squares. Below this, there's a checkbox for 'Show head movement (green for L, red for R)' which is checked, and a 'Visualization' dropdown menu with options 'Default', 'Explore', and 'Blaze'. On the right, there's a 'Machine' section with the following text: 'Machine 1RB1LC 1RC1RB 1RD0LE 1LA1LD 1RZ0LA', 'Wiki entry: 5-state busy beaver winner', 'Status: Decided (Halt)', 'Decider: Cog-BB5', and 'Machine code:'. Below this is a table of machine code instructions:

	0	1
A	1RB	1LC
B	1RC	1RB
C	1RD	0LE
D	1LA	1LD
E	1RZ	0LA

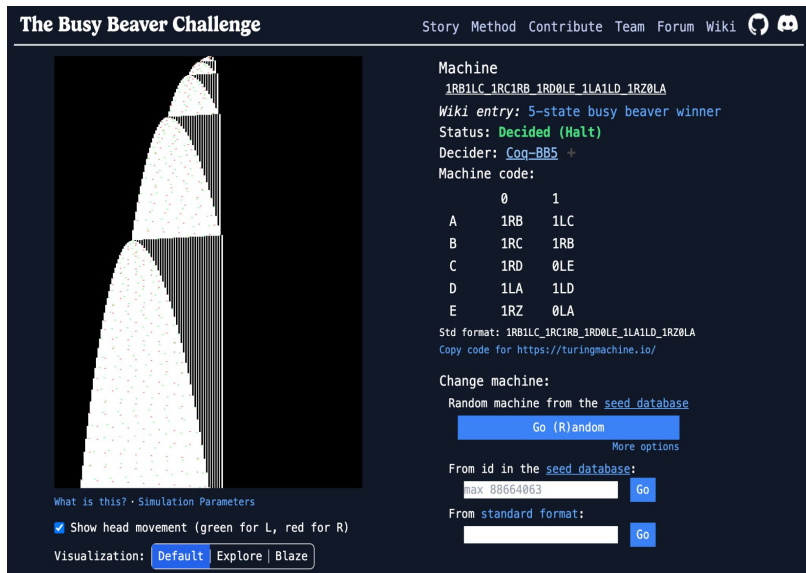
Below the table, it says 'Std format: 1RB1LC 1RC1RB 1RD0LE 1LA1LD 1RZ0LA' and 'Copy code for https://turingmachine.io/'. There's also a 'Change machine:' section with a 'Random machine from the seed database' button, a 'Go (R)andom' button, and a 'More options' link. Below that, there's a 'From id in the seed database:' section with a text input field containing 'max 88664063' and a 'Go' button. Finally, there's a 'From standard format:' section with a text input field and a 'Go' button.



<https://bbchallenge.org>

- **Goal:** collaboratively solving the conjecture “ $BB(5) = 47,176,870$ ”
- **Consists of:** website, Discord server, forum, wiki, *galaxy* of GitHub repositories

# The Busy Beaver Challenge (bbchallenge)



The screenshot shows the 'The Busy Beaver Challenge' website. On the left is a visualization of a busy beaver machine's output, showing a series of vertical bars of increasing height. The right side contains text information about the current machine: 'Machine 1RB1LC 1RC1RB 1RD0LE 1LA1LD 1RZ0LA', 'Wiki entry: 5-state busy beaver winner', 'Status: Decided (Halt)', 'Decider: Coq-BB5', and 'Machine code:'. Below this is a table of states: A (1RB, 1LC), B (1RC, 1RB), C (1RD, 0LE), D (1LA, 1LD), and E (1RZ, 0LA). It also shows the 'Std format' and a link to 'Copy code for https://turingmachine.io/'. At the bottom, there are buttons for 'Go (R)andom' and 'Go' next to input fields for 'From id in the seed database' and 'From standard format'.

- Launched in 2022
- **Goal:** collaboratively solving the conjecture “BB(5) = 47,176,870”
- **Consists of:** website, Discord server, forum, wiki,
- Online, async, almost exclusively communicating on **Discord**
- No management: “**entropically-driven research**”
  - Even the formal proof happened through a grassroots initiative
- 47,000 website unique visitors, ~50 daily
- 1000+ members on Discord
- 50,000+ messages exchanged
- ~50 active contributors
- 19 contributors whose contribution made it to the Coq proof
- *Galaxy* of ~30 GitHub repositories

<https://bbchallenge.org>





## Computer Science &gt; Logic in Computer Science

*[Submitted on 15 Sep 2025]*

# Determination of the fifth Busy Beaver value

The [bbchallenge Collaboration](#): [Justin Blanchard](#), [Daniel Briggs](#), [Konrad Deka](#), [Nathan Fenner](#), [Yannick Forster](#), [Georgi Georgiev](#) (Skelet), [Matthew L. House](#), [Rachel Hunter](#), [Iijil](#), [Maja Kądziołka](#), [Pavel Kropitz](#), [Shawn Ligocki](#), [mxdys](#), [Mateusz Naściszewski](#), [savask](#), [Tristan Stérin](#), [Chris Xu](#), [Jason Yuen](#), [Théo Zimmermann](#)

We prove that  $S(5) = 47, 176, 870$  using the Coq proof assistant. The Busy Beaver value  $S(n)$  is the maximum number of steps that an  $n$ -state 2-symbol Turing machine can perform from the all-zero tape before halting, and  $S$  was historically introduced by Tibor Radó in 1962 as one of the simplest examples of an uncomputable function. The proof enumerates 181, 385, 789 Turing machines with 5 states and, for each machine, decides whether it halts or not. Our result marks the first determination of a new Busy Beaver value in over 40 years and the first Busy Beaver value ever to be formally verified, attesting to the effectiveness of massively collaborative online research ([this http URL](#)).

# Summary: landscape of small BB values

## Legend

Proved in Coq

Existence of a “Cryptid”

Symbols	2-State	3-State	4-State	5-State	6-State
2	$S(2) = 6$ [69]	$S(3) = 21$ [62]	$S(4) = 107$ [10]	$S(5) = 47,176,870$	$S(6) > 2 \uparrow\uparrow\uparrow 5$
3	$S(2, 3) = 38$ [47]	$S(3, 3) > 10^{17}$	$S(4, 3) > 2 \uparrow\uparrow\uparrow 2^{2^{32}}$	—	—
4	$S(2, 4) = 3,932,964$	$S(3, 4) > 2 \uparrow^{15} 5$	—	—	—
5	$S(2, 5) > 10 \uparrow\uparrow 4$	—	—	—	—

# Summary: landscape of small BB values

## Legend

Proved in Coq

Existence of a “Cryptid”

Symbols	2-State	3-State	4-State	5-State	6-State
2	$S(2) = 6$ [69]	$S(3) = 21$ [62]	$S(4) = 107$ [10]	$S(5) = 47,176,870$	$S(6) > 2 \uparrow\uparrow\uparrow 5$
3	$S(2, 3) = 38$ [47]	$S(3, 3) > 10^{17}$	$S(4, 3) > 2 \uparrow\uparrow\uparrow 2^{2^{32}}$	—	—
4	$S(2, 4) = 3,932,964$	$S(3, 4) > 2 \uparrow^{15} 5$	—	—	—
5	$S(2, 5) > 10 \uparrow\uparrow 4$	—	—	—	—

## Antihydra

- 6-state TM
- Simplest open problem in mathematics, on the BB scale

	0	1
A	1RB	1RA
B	0LC	1LE
C	1LD	1LC
D	1LA	0LB
E	1LF	1RE
F	---	0RA

Iterate Collatz-like function starting from 8:

$$f(2n) = 3n$$

$$f(2n + 1) = 3n + 1$$

Do you ever get twice as many more odd terms than even ones?

Antihydra halts iff this is true.

<https://bbchallenge.org/antihydra>