BB(5) in 5 minutes

The bbchallenge Collaboration
bbchallenge.org

Maynooth University
National University of Ireland Maynooth

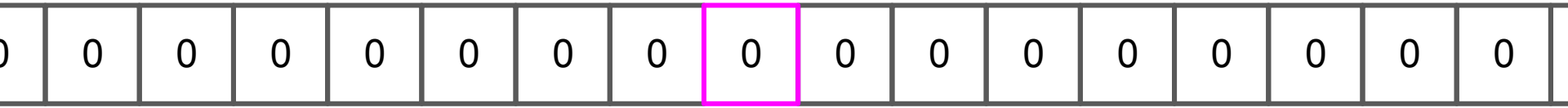
Hamilton Institute

ERC No 772766, SFI 18/ERCS/5746

Does it halt?

	0	1
A	1RB	1LC
B	1RC	1RB
C	1RD	0LE
D	1LA	1LD
E	---	0LA

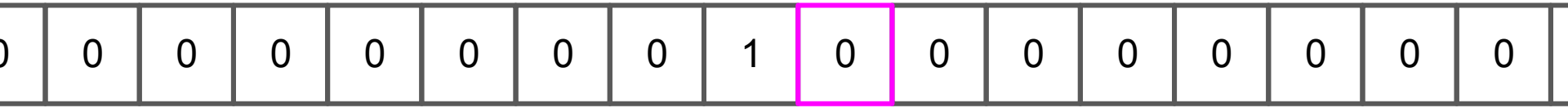
Step #0



Does it halt?

	0	1
A	1RB	1LC
B	1RC	1RB
C	1RD	0LE
D	1LA	1LD
E	---	0LA

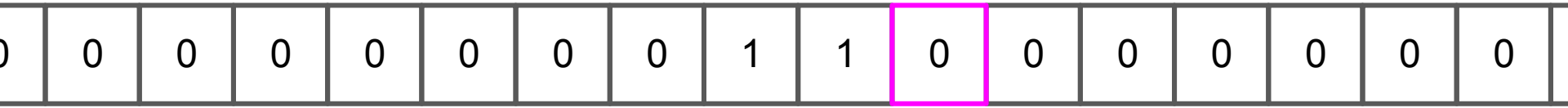
Step #1



Does it halt?

	0	1
A	1RB	1LC
B	1RC	1RB
C	1RD	0LE
D	1LA	1LD
E	---	0LA

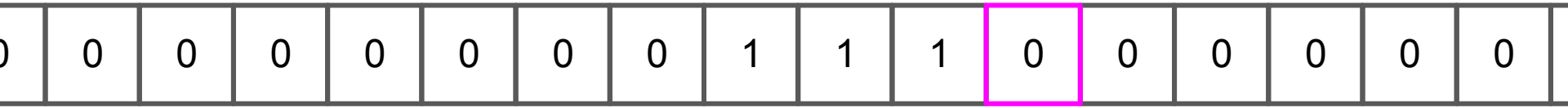
Step #2



Does it halt?

	0	1
A	1RB	1LC
B	1RC	1RB
C	1RD	0LE
D	1LA	1LD
E	---	0LA

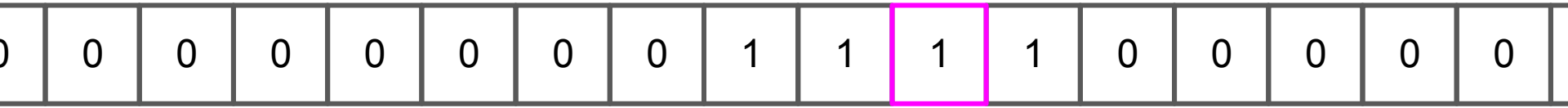
Step #3



Does it halt?

	0	1
A	1RB	1LC
B	1RC	1RB
C	1RD	0LE
D	1LA	1LD
E	---	0LA

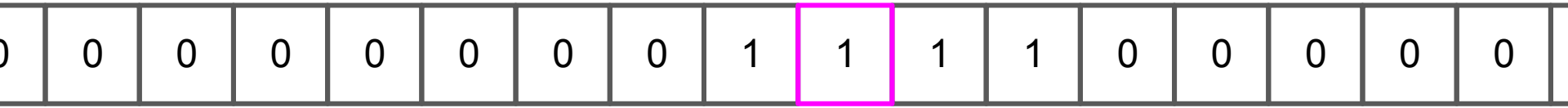
Step #4



Does it halt?

	0	1
A	1RB	1LC
B	1RC	1RB
C	1RD	0LE
D	1LA	1LD
E	---	0LA

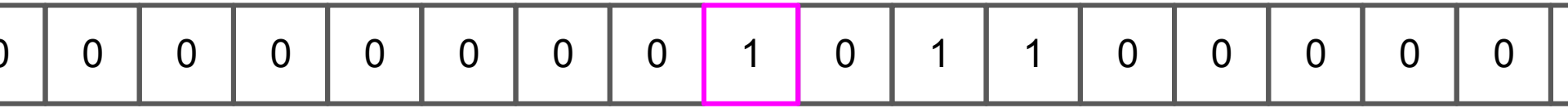
Step #5



Does it halt?

	0	1
A	1RB	1LC
B	1RC	1RB
C	1RD	0LE
D	1LA	1LD
E	---	0LA

Step #6



Does it halt?

	0	1
A	1RB	1LC
B	1RC	1RB
C	1RD	0LE
D	1LA	1LD
E	---	0LA

Step #6

Will we ever read a 0 in state E ?

0	0	0	0	0	0	0	0	1	0	1	1	0	0	0	0	0

Does it halt?

	0	1
A	1RB	1LC
B	1RC	1RB
C	1RD	0LE
D	1LA	1LD
E	---	0LA

Step #6

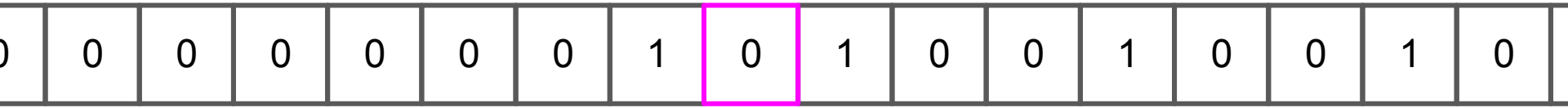
er read a 0 in state E ?

0	0	0	0	0		1	0	0	0	0	0	

Does it halt?
Yes!

	0	1
A	1RB	1LC
B	1RC	1RB
C	1RD	0LE
D	1LA	1LD
E	---	0LA

Step #47,176,870

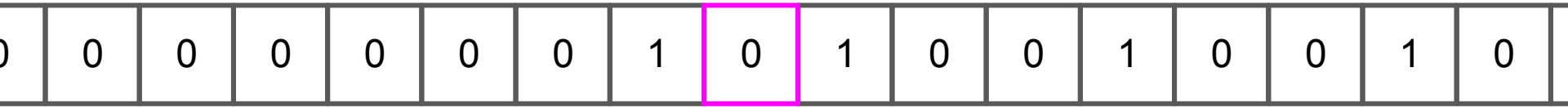


The machine has halted!!

Does it halt?

	0	1
A	1RB	1LC
B	1RC	1RB
C	1RD	0LE
D	1LA	1LD
E	---	0LA

Step #47,176,870



The machine has halted!!

Can another 5-state machine do better?

The Busy Beaver function

BB(n) = “Maximum number of steps done by a halting 2-symbol Turing machine with n states starting from all-0 memory tape”

T. Radó. On Non-computable Functions. *Bell System Technical Journal*, 41(3):877–884. 1962.

BB(n) = “Maximum algorithmic bang for your buck”

UNCOMPUTABLE

Tibor Radó, 1895 - 1965

The Busy Beaver function

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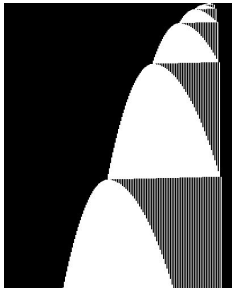
Small busy beaver values:

- BB(1) = 1, BB(2) = 6 [Radó, 1962]
- BB(3) = 21 [Radó and Lin, 1963]
- BB(4) = 107 [Brady, 1983]
- BB(5) ≥ 47,176,870 [Marxen and Buntrock, 1989]

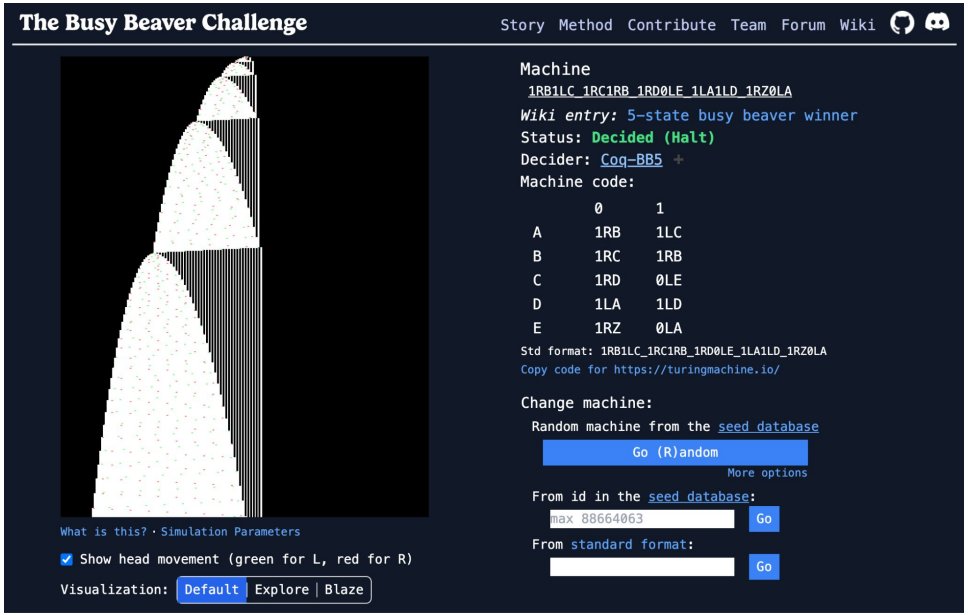
BB(5) = 47,176,870?

Conjectured yes [Aaronson, 2020]



The Busy Beaver Challenge (bbchallenge)

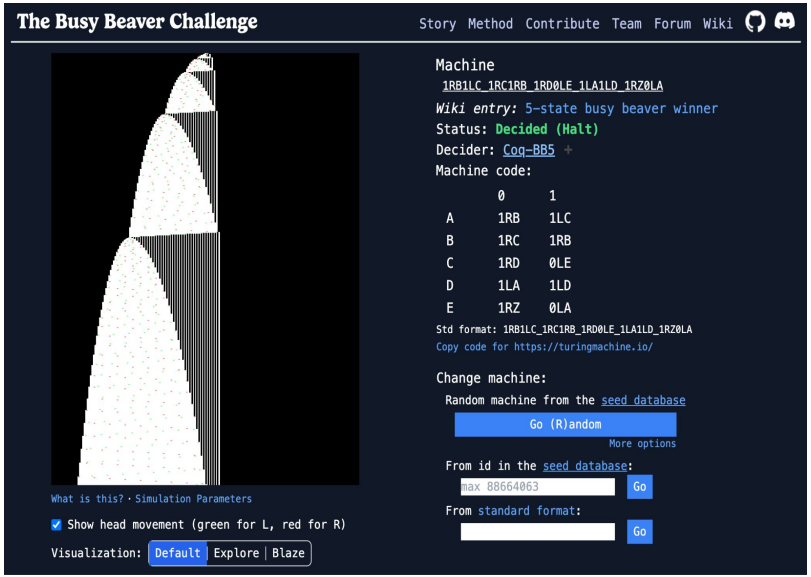
Launched in 2022



https://bbchallenge.org

- **Goal:** collaboratively solving the conjecture “BB(5) = 47,176,870”
- **Consists of:** website, Discord server, forum, wiki, *galaxy* of GitHub repositories

The Busy Beaver Challenge (bbchallenge)



https://bbchallenge.org

- Launched in 2022
- **Goal:** collaboratively solving the conjecture “BB(5) = 47,176,870”
- **Consists of:** website, Discord server, forum, wiki,
- Online, async, almost exclusively communicating on **Discord**
- No management: “**entropically-driven research**”
  - Even the formal proof happened through a grassroot initiative
- 47,000 website unique visitors, ~50 daily
- 1000+ members on Discord
- 50,000+ messages exchanged
- ~50 active contributors
- 19 contributors whose contribution made it to the Coq proof
- *Galaxy* of ~30 GitHub repositories

Computer Science > Logic in Computer Science

[Submitted on 15 Sep 2025]

Determination of the fifth Busy Beaver value

The bbchallenge Collaboration: Justin Blanchard, Daniel Briggs, Konrad Deka, Nathan Fenner, Yannick Forster, Georgi Georgiev (Skelet), Matthew L. House, Rachel Hunter, Iijil, Maja Kądziołka, Pavel Kropitz, Shawn Ligocki, mxdys, Mateusz Naściszewski, savask, Tristan Stérin, Chris Xu, Jason Yuen, Théo Zimmermann

We prove that $S(5) = 47,176,870$ using the Coq proof assistant. The Busy Beaver value $S(n)$ is the maximum number of steps that an n-state 2-symbol Turing machine can perform from the all-zero tape before halting, and S was historically introduced by Tibor Radó in 1962 as one of the simplest examples of an uncomputable function. The proof enumerates $181,385,789$ Turing machines with 5 states and, for each machine, decides whether it halts or not. Our result marks the first determination of a new Busy Beaver value in over 40 years and the first Busy Beaver value ever to be formally verified, attesting to the effectiveness of massively collaborative online research (this http URL).

Summary: landscape of small BB values

Legend

Proved in Coq

Existence of a “Cryptid”

Symbols	2-State	3-State	4-State	5-State	6-State
2	$S(2) = 6$ [69]	$S(3) = 21$ [62]	$S(4) = 107$ [10]	$S(5) = 47{,}176{,}870$	$S(6) > 2 \uparrow\uparrow\uparrow 5$
3	$S(2,3) = 38$ [47]	$S(3,3) > 10^{17}$	$S(4,3) > 2 \uparrow\uparrow\uparrow 2^{2^{32}}$	–	–
4	$S(2,4) = 3{,}932{,}964$	$S(3,4) > 2 \uparrow^{15} 5$	–	–	–
5	$S(2,5) > 10 \uparrow\uparrow 4$	–	–	–	–

Summary: landscape of small BB values

Legend

Proved in Coq

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3	$S(2,3) = 38$ [47]	$S(3,3) > 10^{17}$	$S(4,3) > 2 \uparrow\uparrow\uparrow 2^{2^{32}}$	–	–
4	$S(2,4) = 3{,}932{,}964$	$S(3,4) > 2 \uparrow^{15} 5$	–	–	–
5	$S(2,5) > 10 \uparrow\uparrow 4$	–	–	–	–

Antihydra

- 6-state TM
- Simplest open problem in mathematics, on the BB scale

	0	1
A	1RB	1RA
B	0LC	1LE
C	1LD	1LC
D	1LA	0LB
E	1LF	1RE
F	---	0RA

Iterate Collatz-like function starting from 8:

$$f(2n) = 3n$$
$$f(2n+1) = 3n+1$$

Do you ever get twice as many more odd terms than even ones?

Antihydra halts iff this is true.

https://bbchallenge.org/antihydra