INTRODUCTION TO BOHMIAN MECHANICS
SUMMER TERM 2016

EXERCISE SHEET 6

Exercise 1: *The Heat Equation*
The (here one-dimensional) heat equation describes the change of temperature in a piece of, say, metal. It reads

$$\frac{\partial}{\partial t}T(x,t) = \alpha \Delta T(x,t)\,,$$

where T is the (non-negative) temperature at position $x \in \mathbb{R}$ and time t.
Imagine a one-dimensional, infinitely long piece of metal that at time $t = 0$ has a temperature distribution $\theta(x)$.

a) What does α mean, physically?
b) Solve this equation for the case of the infinitely long metal piece by finding a propagator.
c) Compare the propagator to that of the Schrödinger equation. Discuss.
d) Assume your initial temperature distribution is only nonzero in a bounded set $\Omega \subset \mathbb{R}$, *i.e.* $\operatorname{supp}\theta \subseteq \Omega$. What can you say about the support of T at times $t > 0$? What about the analogous situation for the Schrödinger equation?

Exercise 2: *The Hydrogen Atom*
The Hydrogen atom is the poster child of quantum mechanics, as it showed the predictive power of this theory to a very high degree of accuracy. Its Hamiltonian reads in spherical coordinates

$$H = -\frac{\hbar^2}{2m}\Delta - \frac{e^2}{r}.$$

a) Recall how the eigenvalue problem was solved to give the Hydrogen eigenfunctions. What are the quantum numbers and the energies?
b) The stationary ground state wave function is given by

$$\Psi_{1s}(r,\theta,\varphi) = \frac{\mathrm{e}^{-r/a_0}}{\sqrt{\pi a_0^3}},$$

with the Bohr radius $a_0 = \hbar^2/(me^2)$. What is the wave function $\psi(r,\theta,\varphi,t)$ in the Hydrogen atom at time t for the initial wave function Ψ_{1s}?
c) Now let Ψ_{1s} evolve freely in time. Compute for the time-evolved wave function the expectation values of $X(t)$ and of $X(t)/t$. Compare with $\langle \Psi_{1s}, -\mathrm{i}\hbar\nabla\Psi_{1s}\rangle$.