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Problem sheet 9
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Riemann Surfaces
Solution

Problem 36*

Recall problem 32 and the function φ given there. On the ring areas $U_1 = U(0,T)$ and $U_2 = U(-T/2, T/2)$ the funtion φ (w.r.t. to the chart given by φ) is the identity. Hence φ has no logarithm in both cases. Call φ_1 the function on U_1 and φ_2 the function on U_2. On $W_0 = p(Y(0,T/2) = \varphi^{-1}(A(e^{-\pi T}, 1))$ the two functions agree and we get $\varphi_1\varphi_2^{-1} = 1$. On $W_1 = p(Y(-T/2,0) = p(Y(T/2,T))$ the two functions differ by a factor $\exp(2\pi i\tau)$. Hence we get $\varphi_1 \equiv \varphi_2$ on $U_1 \cap U_2$. If (Sh2) would hold there should be a global section that restricts to φ_1 and φ_2. But every holomorphic function on X is constant and hence we must have $\varphi_1 = c\exp(h_1)$, $\varphi_2 = c\exp(h_2)$ in contradiction to the definition of φ_1, φ_2.

Note that other functions would do as well. Take f_1, f_2 such that no logarithm exists and such that $f_1 f_2^{-1}$ has a logarithm. For example $f_1(z) = (z - \exp(\pi i\tau))(z - \exp(-\pi i\tau))$ and $f_2(z) = (z-1)(z - \exp(-2\pi i\tau))$. More precisely, if (h_{ij}) is a coboundary - $h_{12} = h_2 - h_1$ then $f_1\exp(h_1) = f_2\exp(h_2)$. Hence there is a global constant function f with $c = f|_{U_i} = f_i\exp(h_i) \Rightarrow f_i = \exp(b - h_i)$ for some b with $\exp(b) = c$. Thus if f_i has no logarithm h_{12} is no coboundary and (Sh2) cannot hold.