Unfortunately there have been two mistakes in the solutions of problem sheet 3 on Wednesday 14.11.

Problem 10

b) The exponential function is **not** proper: The preimage $\{2\pi i n : n \in \mathbb{Z}\}$ of the compact set $\{1\}$ is not bounded, hence not compact. However it is a covering map, since the preimage of every small enough open set $U \subset \mathbb{C}^*$ decomposes into a disjoint union of open sets $V_n \subset \mathbb{C}$ ($\exp(z) = \exp(w) \Rightarrow z = w + m\pi, m \in \mathbb{Z} \Rightarrow$ locally one-to-one + holomorphic, hence locally biholomorphic.)

Problem 12

a) Choose $\Lambda_0 = \Lambda \cap K_{2r}, K_r = \{z \in \mathbb{C} : |z| \leq r\}$. Then for all $w \in \Lambda \setminus \Lambda_0, \forall z \in K_r : |w| > 2|z|$. Hence

$$\begin{aligned}
\left| \sum_{w\in\Lambda\setminus\Lambda_0} \frac{1}{(z-w)^2} - \frac{1}{w^2} \right| &\leq \sum_{w\in\Lambda\setminus\Lambda_0} \frac{|-z^2+2zw|}{|w|^2|z-w|^2} \\
&\leq \sum_{w\in\Lambda\setminus\Lambda_0} \frac{|z||2w-z|}{|w|^2||w|-|w|/2|^2} \\
&\leq \sum_{w\in\Lambda\setminus\Lambda_0} \frac{4|z|||2w|+|w/2||}{|w|^4} \\
&\leq \sum_{w\in\Lambda\setminus\Lambda_0} \frac{10r}{|w|^3} \\
&\leq \frac{1}{w_1w_2|\sin(\angle(w_1,w_2))|} \int_{\mathbb{C}\setminus K_{2r}} \frac{10r}{|w|^3} \mathrm{d}w \\
&= \frac{10r}{w_1w_2|\sin(\angle(w_1,w_2))|} \int_{2r}^{\infty} \int_0^{2\pi} \frac{r}{r^3} \mathrm{d}\varphi \mathrm{d}r \\
&= \frac{20r\pi}{w_1w_2|\sin(\angle(w_1,w_2))|} \left(-0 + \frac{1}{2r}\right) \\
&\leq \frac{10r\pi}{rw_1w_2|\sin(\angle(w_1,w_2))|} \\
&= \frac{10\pi}{w_1w_2|\sin(\angle(w_1,w_2))|} \\
&< \infty.
\end{aligned}$$

Thus $\sum_{w\in\Lambda\setminus\Lambda_0} \frac{1}{(z-w)^2} - \frac{1}{w^2}$ converges uniformely on K_r.