LMU LUDWIG-MAXIMILIANS-UNIVERSITÄT MÜNCHEN

MATHEMATISCHES INSTITUT

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Gewöhnliche Differentialgleichungen

Blatt 6

Aufgabe 1. Let the system

$$\dot{x}_1(t) = x_1(t),$$
$$\dot{x}_2(t) = x_1(t) + x_2(t)$$

with $x_1(0) = a$ and $x_2(0) = b$. Show that its unique solution is the curve

$$x(t) = \big(ae^t, e^t(at + b)\big).$$

Aufgabe 2. Find $A \in M_2(\mathbb{R})$ such that the curve

$$x(t) = \big(e^{2t} - e^{-t}, e^{2t} + 2e^{-t}\big)$$

is one solution to the system

$$\dot{x}(t) = Ax(t).$$

Aufgabe 3. Let $A \in M_n(\mathbb{R})$ with n distinct, real eigenvalues $\lambda_1, \dots, \lambda_n$. Find a condition P on $\lambda_1, \dots, \lambda_n$ such that the following are equivalent:
(i) $P(\lambda_1, \dots, \lambda_n)$.
(ii) For every non-zero solution $x(t)$ to the system $\dot{x}(t) = Ax(t)$ it holds

$$\lim_{t \to +\infty} |x(t)| = +\infty.$$

(You also need to show that (i) and (ii) are equivalent.)

Aufgabe 4. Prove Theorem 1.4.19.

Abgabe. Donnerstag, 07. Juni 2018 in der Vorlesung.

Besprechung. Montag, 11. Juni 2018, in der Übung.