LMU | LUDWIG-MAXIMILIANS-UNIVERSITÄT MÜNCHEN | MATHEMATISCHES INSTITUT

Prof. Dr. Hans-Dieter Donder
Iosif Petrakis

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Modelle der Mengenlehre

Exam

Family Name: ______________ First name: ______________

Student ID: ______________ Term: ______________

Degree course: Bachelor, PO ❏ 2007 ❏ 2010 ❏ 2011 Master, PO ❏ 2010 ❏ 2011

Lehramt Gymnasium: ❏ modularisiert ❏ nicht modularisiert

❏ Diplom ❏ Other: ______________

Major subject: ❏ Mathematik ❏ Wirtschaftsm. ❏ Inf. ❏ Phys. ❏ Stat. ❏ ______

Minor subject: ❏ Mathematik ❏ Wirtschaftsm. ❏ Inf. ❏ Phys. ❏ Stat. ❏ ______

Credit Points to be used for ❏ Hauptfach ❏ Nebenfach (Bachelor / Master)

Please switch off your mobile phone and do not place it on the table; place your identity and student ID cards on the table so that they are clearly visible.
Please check that you have received all **ten problems**.
Please do not write with the colors red or green. Write **on every page** your **family name and your first name**.
Please make sure to submit only one solution for each problem; cross out everything that should not be graded.
If you need a tutorial certificate (Übungsschein / nicht modularisiert) please fill in the form on the second next page.
By entering a pseudonym (e.g. the last four digits of your student ID number) in the appropriate box on the left at the top of the next page you will give your permission to the publication of your results on the lecture's homepage.

You have **120 minutes** in total to complete this examination.

Good luck!

Pseudonym	1	2	3	4	5
	/4	/4	/4	/4	/4

	6	7	8	9	10	$\sum$
	/4	/4	/4	/4	/4	/40

Please read carefully each of the following statements, decide whether it is true or false and tick your answer accordingly.

Each correct answer gives one point. Each false answer gives zero points. The optimal total sum is 40 points.

Exercise 1: **a.** The axiom scheme of Replacement is the following scheme

$$\forall_x \exists_v \forall_y (y \in v \leftrightarrow \exists_z (z \in x \wedge \phi(z, y, \vec{w})).$$

❑ True ❑ False

b. If F is a function and if $F''A \notin V$, then $A \notin V$.

❑ True ❑ False

c. If $n > 0$, then ZF $\vdash \exists x_0, x_1, \ldots, x_n (\bigwedge_{i=1}^{n} (x_{i-1} \in x_i) \wedge (x_n \in x_0))$.

❑ True ❑ False

d. The intersection of two proper classes is a proper class.

❑ True ❑ False

Exercise 2: **a.** If $\bigcup u \subseteq u$, then u is transitive.

❑ True ❑ False

b. A set is an ordinal number if and only if it is a transitive set of transitive sets.

❑ True ❑ False

c. There is no bijection between the open unit interval $(0, 1)$ and $\mathbb{R}$ that takes rationals to rationals and irrationals to irrationals.

❑ True ❑ False

d. Assume that a is a finite set and $f : a \to a$. Then f is an injection if and only if $\text{rng}(f) = a$.

❑ True ❑ False

Exercise 3: **a.** There is an ordinal α such that $V_\alpha \notin V$.

❏ True ❏ False

b. If $F : \mathrm{On} \to \mathrm{On}$ is increasing i.e., $\forall_{\alpha,\beta\in\mathrm{On}}(\alpha < \beta \to F(\alpha) < F(\beta))$, then $F(\lambda) = \bigcup_{\alpha<\lambda} F(\alpha)$, for every limit ordinal λ.

❏ True ❏ False

c. If $F : \mathrm{On} \to \mathrm{On}$ satisfies the property $F(\lambda) = \bigcup_{\alpha<\lambda} F(\alpha)$, for every limit ordinal λ, then F is increasing.

❏ True ❏ False

d. If F is increasing, then $\forall_{\alpha,\beta\in\mathrm{On}}(F(\alpha) < F(\beta) \to \alpha < \beta)$.

❏ True ❏ False

Exercise 4: **a.** If F is a function such that $n \mapsto V_{\omega+n}$, then $F''\omega \in V_{\omega\cdot 2}$.

❏ True ❏ False

b. The limit ordinals form a closed and unbounded class.

❏ True ❏ False

c. "x is a successor ordinal" is not equivalent to a Σ_0-formula.

❏ True ❏ False

d. "x is a limit ordinal" is equivalent to a Σ_0-formula.

❏ True ❏ False

Exercise 5: **a.** $\mathrm{cf}(\omega + \omega) = \omega$.

❏ True ❏ False

b. $\mathrm{cf}(\omega^2) > \omega$.

❏ True ❏ False

c. $\omega_{\alpha+1}$ is regular, for every ordinal α.

❏ True ❏ False

d. If $\lambda < \mathrm{cf}(\kappa)$ and $f : \lambda \to \kappa$, then $\mathrm{rng}(f)$ is bounded in κ.

❏ True ❏ False

Exercise 6: **a.** $V = L \to \text{AC} + \text{GCH}$.

❑ True ❑ False

b. L is an inner model of ZF.

❑ True ❑ False

c. If $V = \text{HOD}$, then $V = \text{OD}$.

❑ True ❑ False

d. If $a = \{2n \mid n \in \omega\}$, then $a \in \text{OD}$.

❑ True ❑ False

Exercise 7: **a.** $L_\omega \subsetneq V_\omega$.

❑ True ❑ False

b. $\text{Def}(\omega) = \mathcal{P}(\omega)$.

❑ True ❑ False

c. If $\alpha > \omega$ is a limit ordinal, then $L_\alpha \models ZF^-$.

❑ True ❑ False

d. V is not an inner model of ZF.

❑ True ❑ False

Exercise 8: W is an inner model of ZF.

a. If $\text{ZF} \vdash \phi$, then $\text{ZF} \vdash \phi^W$.

❑ True ❑ False

b. If $\text{ZF} \vdash \phi^W$ and ZF is consistent, then $\text{ZF} + \phi$ is consistent.

❑ True ❑ False

c. If $\phi(\vec{x})$ is Π_1, then $\forall_{\vec{x} \in W}(\phi(\vec{x}) \leftarrow \phi^W(\vec{x}))$.

❑ True ❑ False

d. If $\phi(\vec{x})$ is Σ_1, then $\forall_{\vec{x} \in W}(\phi(\vec{x}) \to \phi^W(\vec{x}))$.

❑ True ❑ False

Exercise 9: Let M be a countable transitive model of ZFC, $<\mathbb{P}, \leq, \mathbb{1}> \in M$ a set of conditions, G is M-generic, and $\Vdash$ the corresponding forcing relation.

a. $p \Vdash \forall x \phi$ if and only if $\forall a \in M^{\mathbb{P}} p \Vdash \phi(a)$.

❑ True ❑ False

b. $p \Vdash \phi \vee \psi$ if and only if $\forall q \leq p \exists r \leq q (r \Vdash \phi \vee r \Vdash \psi)$.

❑ True ❑ False

c. If $E \subseteq \mathbb{P}$ and $E \in M$, then $G \cap E \neq \emptyset$ or $\exists g \in G\ \forall e \in E(e, g$ are incompatible).

❑ True ❑ False

d. Let $\mathbb{P} = \{p \in M \mid p : n \to \omega, n \in \omega\}$, $<\mathbb{P}, \supseteq, \emptyset>$ is the conditions set, $G \subseteq \mathbb{P}$ is M-generic and $f = \bigcup G : \omega \to \omega$. Then for every $g \in M$ there exists $n \in \omega$ such that $f(n) > g(n)$.

❑ True ❑ False

Exercise 10: Suppose that M is a countable transitive model for ZFC, $< \mathbb{P}, \supseteq \emptyset >$, where $\mathbb{P} = \{p : n \to \{0, 1, 2, 3\} \mid n \in \omega\}$, $G \subseteq \mathbb{P}$ is M-generic and Φ is a name for $\bigcup G$.

a. $\emptyset \Vdash \mathrm{dom}(\Phi) = \omega$.

❑ True ❑ False

b. $\emptyset \Vdash \hat{3} \in \mathrm{rng}(\Phi)$.

❑ True ❑ False

c. There is some $p \in \mathbb{P}$ such that $p \Vdash$ (Φ takes the value $\hat{3}$ exactly 3 times).

❑ True ❑ False

d. $\emptyset \Vdash \Phi \notin L$.

❑ True ❑ False