LUDWIG-MAXIMILIANS-UNIVERSITÄT MÜNCHEN

MATHEMATISCHES INSTITUT

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Modelle der Mengenlehre

Answers

Exercise 1: **a.** The axiom scheme of Replacement is the following scheme

$$\forall_x \exists_v \forall_y (y \in v \leftrightarrow \exists_z (z \in x \wedge \phi(z, y, \vec{w})).$$

❏ False

b. If F is a function and if $F''A \notin V$, then $A \notin V$.

❏ True

c. If $n > 0$, then ZF $\vdash \exists x_0, x_1, \ldots, x_n (\bigwedge_{i=1}^{n} (x_{i-1} \in x_i) \wedge (x_n \in x_0))$.

❏ False

d. The intersection of two proper classes is a proper class.

❏ False

Exercise 2: **a.** If $\bigcup u \subseteq u$, then u is transitive.

❏ True

b. A set is an ordinal number if and only if it is a transitive set of transitive sets.

❏ True

c. There is no bijection between the open unit interval $(0, 1)$ and $\mathbb{R}$ that takes rationals to rationals and irrationals to irrationals.

❏ False

d. Assume that a is a finite set and $f : a \to a$. Then f is an injection if and only if $\text{rng}(f) = a$.

❏ True

Exercise 3: **a.** There is an ordinal α such that $V_\alpha \notin V$.

❑ False

b. If $F : \mathrm{On} \to \mathrm{On}$ is increasing i.e., $\forall_{\alpha,\beta\in\mathrm{On}}(\alpha < \beta \to F(\alpha) < F(\beta))$, then $F(\lambda) = \bigcup_{\alpha<\lambda} F(\alpha)$, for every limit ordinal λ.

❑ False

c. If $F : \mathrm{On} \to \mathrm{On}$ satisfies the property $F(\lambda) = \bigcup_{\alpha<\lambda} F(\alpha)$, for every limit ordinal λ, then F is increasing.

❑ False

d. If F is increasing, then $\forall_{\alpha,\beta\in\mathrm{On}}(F(\alpha) < F(\beta) \to \alpha < \beta)$.

❑ True

Exercise 4: **a.** If F is a function such that $n \mapsto V_{\omega+n}$, then $F''\omega \in V_{\omega\cdot 2}$.

❑ False

b. The limit ordinals form a closed and unbounded class.

❑ True

c. "x is a successor ordinal" is not equivalent to a Σ_0-formula.

❑ False

d. "x is a limit ordinal" is equivalent to a Σ_0-formula.

❑ True

Exercise 5: **a.** $\mathrm{cf}(\omega + \omega) = \omega$.

❑ True

b. $\mathrm{cf}(\omega^2) > \omega$.

❑ False

c. $\omega_{\alpha+1}$ is regular, for every ordinal α.

❑ True

d. If $\lambda < \mathrm{cf}(\kappa)$ and $f : \lambda \to \kappa$, then $\mathrm{rng}(f)$ is bounded in κ.

❑ True

Exercise 6: **a.** $V = L \rightarrow \text{AC} + \text{GCH}$.

❏ True

b. L is an inner model of ZF.

❏ True

c. If $V = \text{HOD}$, then $V = \text{OD}$.

❏ True

d. If $a = \{2n \mid n \in \omega\}$, then $a \in \text{OD}$.

❏ True

Exercise 7: **a.** $L_\omega \subsetneq V_\omega$.

❏ False

b. $\text{Def}(\omega) = \mathcal{P}(\omega)$.

❏ False

c. If $\alpha > \omega$ is a limit ordinal, then $L_\alpha \models ZF^-$.

❏ False

d. V is not an inner model of ZF.

❏ False

Exercise 8: W is an inner model of ZF.

a. If $\text{ZF} \vdash \phi$, then $\text{ZF} \vdash \phi^W$.

❏ True

b. If $\text{ZF} \vdash \phi^W$ and ZF is consistent, then $\text{ZF} + \phi$ is consistent.

❏ True

c. If $\phi(\vec{x})$ is Π_1, then $\forall_{\vec{x} \in W}(\phi(\vec{x}) \leftarrow \phi^W(\vec{x}))$.

❏ False

d. If $\phi(\vec{x})$ is Σ_1, then $\forall_{\vec{x} \in W}(\phi(\vec{x}) \rightarrow \phi^W(\vec{x}))$.

❏ False

Exercise 9: Let M be a countable transitive model of ZFC, $<\mathbb{P}, \leq, \mathbb{1}> \in M$ a set of conditions, G is M-generic, and $\Vdash$ the corresponding forcing relation.

a. $p \Vdash \forall x \phi$ if and only if $\forall a \in M^{\mathbb{P}} p \Vdash \phi(a)$.

❑ True

b. $p \Vdash \phi \vee \psi$ if and only if $\forall q \leq p \exists r \leq q(r \Vdash \phi \vee r \Vdash \psi)$.

❑ True

c. If $E \subseteq \mathbb{P}$ and $E \in M$, then $G \cap E \neq \emptyset$ or $\exists g \in G\ \forall e \in E(e, g$ are incompatible).

❑ True

d. Let $\mathbb{P} = \{p \in M \mid p : n \to \omega, n \in \omega\}$, $<\mathbb{P}, \supseteq, \emptyset>$ is the conditions set, $G \subseteq \mathbb{P}$ is M-generic and $f = \bigcup G : \omega \to \omega$. Then for every $g \in M$ there exists $n \in \omega$ such that $f(n) > g(n)$.

❑ True

Exercise 10: Suppose that M is a countable transitive model for ZFC, $< \mathbb{P}, \supseteq \emptyset >$, where $\mathbb{P} = \{p : n \to \{0, 1, 2, 3\} \mid n \in \omega\}$, $G \subseteq \mathbb{P}$ is M-generic and Φ is a name for $\bigcup G$.

a. $\emptyset \Vdash \mathrm{dom}(\Phi) = \omega$.

❑ True

b. $\emptyset \Vdash \hat{3} \in \mathrm{rng}(\Phi)$.

❑ True

c. There is some $p \in \mathbb{P}$ such that $p \Vdash$ (Φ takes the value $\hat{3}$ exactly 3 times).

❑ False

d. $\emptyset \Vdash \Phi \notin L$.

❑ True