Conference “Mathematical Physics and PDEs”

Herrsching, December 15-20, 2024

Program

Monday:

9:00-9:40 Jan Philip Solovej
9:45-10:25 Geneviève Dusson

10:30-11:00 Break

11:00-11:40 Wilhelm Zwerger
11:45-12:00 Mirco Piccinini
12:05-12:20 Jack Thomas

14:00-14:40 Ulrich Schollwöck
14:45-15:25 Isabelle Gallagher

15:30-16:00 Break

16:00-16:40 Nicolas Rougerie
16:45-17:45 Poster pitch talks

20:00-22:00 Posters

Tuesday:

9:00-9:40 Maria Esteban
9:45-10:25 Søren Fournais

10:30-11:00 Break

11:00-11:40 Jani Lukkarinen
11:45-12:00 Aleksis Vuoksenmaa
12:05-12:20 Stefano Rossi

14:00-14:40 Mikaela Iacobelli
14:45-15:25 Nicolas Visciglia

15:30-16:00 Break

16:00-16:40 Antti Knowles

16:45-17:25 Julien Sabin

Wednesday:

9:00-9:40 Michael Loss
9:45-10:25 Cyrill Muratov

10:30-11:00 Break

11:00-11:40 Benjamin Schlein
11:45-12:25 Giacomo De Palma

Free afternoon - social activities
Conference dinner

Thursday:

9:00-9:40 Luis Vega
9:45-10:25 Gigliola Staffilani

10:30-11:00 Break

11:00-11:40 Toan Nguyen
11:45-12:00 Yahui Li
12:05-12:20 Asbjørn Lauritsen

14:00-14:40 Enno Lenzmann
14:45-15:25 Simona Rota Nodari

15:30-16:00 Break

16:00-16:40 Andrea R. Nahmod
16:45-17:25 Eric Carlen

Friday:

9:00-9:40 Étienne Sandier
9:45-10:25 Frank Pollmann

10:30-11:00 Break

11:00-11:40 Lea Bossmann
11:45-12:25 Robert Seiringer

Titles and abstracts

Lea Bossmann (Erlangen): Focusing dynamics for 2d Bose gases in the instability regime

Abstract: We consider the dynamics of a 2d Bose gas with singular attractive interactions in the instability regime, where the corresponding focusing nonlinear Schrödinger equation (NLS) has a blow-up. We show that the evolution of the condensate is effectively described by this NLS for all times before the blow-up. Moreover, we prove the validity of the Bogoliubov approximation for the fluctuation dynamics, resulting in a norm approximation of the many-body dynamics. This is joint work with Charlotte Dietze and Phan Thành Nam.

Eric Carlen (Rutgers)**:** Boundary driven quantum systems and a Mehler formula for fermions

There is much interest in non equilibrium stationary states for quantum spin chains that interact with reservoirs modeled by Lindblad equations, just as in a classical system one can model a heat bath by coupling an edge to a driving Langevin equation. The quantum situation is very different from the classical situation, in which if there is only one thermal bath coupled in this way, the steady state can be the Gibbs equilibrium state at a temperature determined by the driving Langevin equation. Quantum mechanically, this is not possible. We consider system evolving under a global Hamiltonian dynamics, coupled to a Lindblad equation acting on a few sites. Unless the Hamiltonian is fairly trivial, no matter how the Lindbladian is chosen, the Gibbs state will not be a steady state. We present some no-go theorems of this type that are recent work with Joel Lebowitz and David Huse. Continuing this investigation, we then go on to look at the nature of steady state in some specific models with small numbers of particles, finding currents in the steady state, even when there is coupling to only one reservoir. For this, we write the systems in terms of fermions, and use a Mehler type formula for fermions to facilitate the computations. Note that term "Mehler formula" is for at least two things, one being a formula for a Gaussian integral kernel solving a Langevin equation, But in Mehler's 1866 paper, there is another type of formula involving rotations in a doubled space of variables, and the fermionic version of this leads directly to a Stinespring dilation of the semigroup solving certain Linblad evolution equations which is what we use here.

Geneviève Dusson (Bourgogne): A multipoint perturbation for eigenvalue problems

Abstract: In standard perturbation theory, approximate solutions to a modified problem are obtained in the neighbourhood of a known solution to a reference problem. In the context of quantum mechanics, this approach is commonly used to approximate the eigenmodes of a perturbed Hamiltonian based on those of a reference Hamiltonian. In this talk, I will introduce a multipoint perturbation method that leverages multiple nearby solutions simultaneously to achieve more accurate approximations than conventional perturbation theory. I will illustrate how this method can be used to accelerate ab initio molecular dynamics simulations.

Maria Esteban (Paris Dauphine)**:** Some conjectures concerning the ground state energy of a relativistic electron in a molecular configuration.

Abstract: In this talk I will present some results and open problems concerning the lowest eigenvalue of a Dirac operator with a general muti-pole external electrostatic potential. It describes a relativistic quantum electron moving in the field of some (pointwise or extended) nuclei, possibly in a molecule. One of the main questions we ask is whether the eigenvalue is minimal when the nuclear charge is concentrated at one single point. This well-known property in non-relativistic quantum mechanics has escaped all attempts of proof in the relativistic case. This is mainly work in collaboration with M. Lewin and E. Séré, involving also recent discussions with Michael Loss.

Søren Fournais (Copenhagen): On the tunnelling effect with magnetic fields.

Abstract: The mathematical analysis of the tunnelling effect between electric wells was a key milestone in semiclassical analysis reached in the 1980’s with important contributions by Simon and Helffer-Sjöstrand. However, the analysis of the effect in the presence of magnetic fields was largely left open. Technically, this is because the complex phases that a magnetic field induces in the eigenfunctions causes cancellations that are delicate to control to the required precision. Although a general theory of magnetic tunnelling is still lacking at the level of generality of the non-magnetic theory, there has been important progress in recent years, in particular in low-dimensional situations or in special geometries. In this talk I will present some of the magnetic results. Results are in particular in collaboration with Yannick Guedes Bonthonneau, Leo Morin and Nicolas Raymond.

Isabelle Gallagher (Paris Diderot): On the dynamics of dilute gases

Abstract: The evolution of a gas can be described by different models depending on the scale of observation. A natural question, raised by Hilbert in his sixth problem, is whether these models provide consistent predictions. In the case of gases of hard spheres, Lanford showed in 1974 that the Boltzmann equation appears as a law of large numbers in the low density limit, at least for very short times. In this talk we will present Lanford's result, and time permitting, some more recent extensions to understand fluctuations and large deviations around the Boltzmann equation. These are joint works with Thierry Bodineau, Laure Saint-Raymond and Sergio Simonella.

Mikaela Iacobelli (Zurich): Stability and singular limits in plasma physics

Abstract: In this talk, we will present two kinetic models that are used to describe the evolution of charged particles in plasmas: the Vlasov-Poisson system and the Vlasov-Poisson system with massless electrons. These systems model respectively the evolution of electrons, and ions in a plasma. We will discuss the well-posedness of these systems, the stability of solutions, and their behavior under singular limits. Finally, we will introduce a new class of Wasserstein-type distances specifically designed to tackle stability questions for kinetic equations.

Antti Knowles (Geneva): Euclidean field theories and interacting Bose gases

Abstract: Euclidean field theories have been extensively studied in the mathematical literature since the sixties, motivated by high-energy physics and statistical mechanics. Formally, such a theory is given by a Gibbs measure associated with a Euclidean action functional over a space of distributions. In this talk I explain how some such theories arise as high-density limits of interacting Bose gases at positive temperature. This provides a rigorous derivation of them starting from a realistic microscopic model of statistical mechanics. I focus on field theories with a quartic, local or nonlocal, interaction in dimensions ≤ 3. Owing to the singularity of the Gaussian free field in dimensions higher than one, the interaction is ill-defined and has to be renormalized by infinite mass and energy counterterms. The proof is based on a functional integral representation of the interacting Bose gas. Based on joint work with Cristina Caraci, Jurg Fröhlich, Alessio Ranallo, Benjamin Schlein, Vedran Sohinger, and Pedro Torres Giesteira.

Enno Lenzmann (Basel): Continuum Calogero-Moser Systems: Recent Results and Open Questions

Abstract: This is an expository talk about a newly discovered class of completely integrable Hamiltonian PDEs, which arise as (formal) continuum limits of discrete classical Calogero-Moser systems. A particularly intriguing feature of these "continuum CS-systems" is their delicate interplay between scaling-criticality and their complete integrable nature. I will review the current state of affairs about well-posedness, blowup, and soliton dynamics with a strong focus on future open problems. Parts of this talk is based on joint work with P.~Gérard (Paris-Saclay).

Michael Loss (Georgia Tech): Sharp Inequalities for Spinors

Abstract: While functional inequalities play an important role in mathematical physics, the focus was usually on Schr\"odinger operators with some notable exceptions like Hardy inequalities for Dirac operators. In this talk I present some preliminary results for the sharp spinorial version of the Caffarelli-Kohn-Nirenberg inequalities, i.e., Sobolev inequalities with weights. This is joint work with Jean Dolbeault, Maria Esteban and Rupert Frank.

Jani Lukkarinen (Helsinki): Estimation of generation and propagation of chaos via cumulant hierarchies

Abstract: Propagation and generation of "chaos" is an important ingredient for rigorous control of applicability of kinetic theory, in general. Chaos is here understood as sufficient statistical independence of random variables related to the "kinetic" observables of the system. Cumulant hierarchy of these random variables thus often gives a way of controlling the evolution and degree of such independence, i.e., the degree of "chaos" in the system. In this talk, we will consider two, qualitatively different, example cases for which kinetic theory is believed to be applicable: the stochastic Kac model with random velocity exchange and the discrete nonlinear

Schrodinger evolution (DNLS) with suitable random, spatially homogeneous initial data. In both cases, we set up suitable random variables and propose methods to control the evolution of their cumulant hierarchies. The talk is based on joint works with Sakari Pirnes and Aleksis Vuoksenmaa, and earlier works with Matteo Marcozzi, Alessia Nota, and Herbert Spohn.

Cyrill Muratov (Pisa): Magnetic skyrmions of higher degree

Abstract: I will present a brief overview of the results on existence and asymptotic properties of magnetic skyrmions — particle-like topologically nontrivial two-dimensional spin textures that have been envisioned as bit encoding states in the emergent field of spintronics. Mathematically these are defined as topologically nontrivial maps of degree +1 from the plane to a sphere which minimize a micromagnetic energy containing the exchange, perpendicular magnetic anisotropy and interfacial Dzyaloshinskii-Moriya interaction (DMI) terms. In ultrathin films, the stray field energy simply renormalizes the anisotropy constant at leading order, but in finite samples it also produces additional non-trivial contributions at the sample edges, promoting nontrivial spin textures. Starting with the whole space problem, I will first discuss the existence of single skyrmions as global energy minimizers at sufficiently small DMI strength. Then, using the quantitative rigidity of the harmonic maps I will present the asymptotic characterization of single skyrmion profiles both in infinite and finite samples. Lastly, I will touch upon the question of existence of multi-skyrmion solutions as minimizers with higher topological degree and present recent existence results obtained jointly with T. Simon and V. Slastikov.

Andrea Nahmod (Amherst): Probabilistic scaling, propagation of randomness and invariant Gibbs measures.

Abstract: In this talk, we will start by describing how classical tools from probability offer a robust framework to understand the dynamics of waves via appropriate ensembles on phase space rather than particular microscopic dynamical trajectories. We will continue by explaining the fundamental shift in paradigm that arises from the “correct” scaling in this context and how it opened the door to unveil the random structures of nonlinear waves that live on high frequencies and fine scales as they propagate. We will then discuss how these ideas broke the logjam in the study of the Gibbs measures associated to nonlinear Schrödinger equations in the context of equilibrium statistical mechanics and of the hyperbolic Φ^4_3 model in the context of constructive quantum field theory. We will end with some open challenges about the long-time propagation of randomness and out-of-equilibrium dynamics.

Toan T. Nguyen (Pennsylvania): Landau damping below survival threshold

Abstract: Landau damping is a classical subject in plasma physics that predicts relaxation to neutrality of charged particles or decay of the self-consistent electric field in a non-equilibrium state. It was recently discovered that there is in fact a non-trivial survival threshold of wave numbers, below which the generating electric field oscillates and disperses like a pure Klein-Gordon wave, known in the physical literature as plasma oscillations or Langmuir’s oscillatory waves, and above which the electric field is exponentially damped due to phase mixing. This talk will present a new framework to study nonlinear particle-wave interaction and to resolve nonlinear Landau damping below survival threshold.

Simona Rota Noradi (Nice): On a nonlinear Schrödinger equation: uniqueness, non-degeneracy and applications

Abstract: In this talk, I will first give a general result on the uniqueness and the non-degeneracy of positive radial solutions to equations of the form $-\Delta u=g(u)$. Then, I will consider the case of the double power nonlinearity $g(u)=u^q-u^p-\mu u$ for $p>q>1$ and $\mu>0$. In this case, the non-degeneracy of the unique positive solution u_μ allows us to derive its behavior in certain regimes of the parameter μ. In particular, this gives some important information about the uniqueness of energy minimizers with fixed mass. Joint work with Mathieu Lewin.

Giacomo De Palma (Bologna): Trained quantum neural networks are Gaussian processes

Abstract: We study quantum neural networks made by parametric one-qubit gates and fixed two-qubit gates in the limit of infinite width, where the generated function is the expectation value of the sum of single-qubit observables over all the qubits. First, we prove that the probability distribution of the function generated by the untrained network with randomly initialized parameters converges in distribution to a Gaussian process whenever each measured qubit is correlated only with few other measured qubits. Then, we analytically characterize the training of the network via gradient descent with square loss on supervised learning problems. We prove that, as long as the network is not affected by barren plateaus, the trained network can perfectly fit the training set and that the probability distribution of the function generated after training still converges in distribution to a Gaussian process. Finally, we consider the statistical noise of the measurement at the output of the network and prove that a polynomial number of measurements is sufficient for all the previous results to hold and that the network can always be trained in polynomial time.

Frank Pollmann (TU Munich): Entanglement Transitions in Unitary Circuit Games

Abstract: Repeated projective measurements in unitary circuits can lead to an entanglement phase transition when the measurement rate is tuned. In this talk, I will discuss a related but distinct scenario: a one-dimensional unitary circuit game, where projective measurements are replaced by dynamically chosen unitary gates designed to compete over entanglement. Two players act on randomly assigned bonds—an "entangler," applying random local unitary gates to generate volume-law entanglement, and a "disentangler," strategically selecting unitary gates, based on limited state information, to constrain the system to area-law entanglement. We investigate the dynamics of this game in three settings: (i) Clifford circuits, (ii) Matchgate circuits, and (iii) generic U(4) unitary circuits. Each setting exhibits qualitatively distinct behaviors, shedding light on different entanglement transition mechanisms. We draw connections to stochastic classical models, offering a deeper perspective on the underlying physics of the entanglement transition.

Nicolas Rougerie (Lyon): Semi-classical limit of Hartree's equation in large magnetic fields

Abstract: We study the dynamics of two dimensional fermionic particles submitted to a perpendicular magnetic field. We start from the Hartree equation for the first reduced density matrix, describing the mean field behaviour of a large fermionic system, and derive a drift transport equation for the particle density. This derivation has been considered previously as a combination of a 'semi-classical limit on the position/momentum phase-space' and a 'gyrokinetic limit of Vlasov's equation'. Here we consider the truly 'large magnetic field regime' where the gap between Landau levels is of the same order as the other energy contributions. The aforementioned approach does not apply and a real 'semi-classical limit on the Landau level index/position of cyclotron orbit center' is called for, as considered in works of Lieb-Solovej-Yngvason and Fournais-Madsen at the level of ground states. We rely on vortex coherent states and the associated Husimi functions to derive the effective dynamics. Joint work with Denis Périce.

Julien Sabin (Rennes): The Hartree equation at positive density: a review

Abstract: This talk will be devoted to a review of results concerning the time-dependent Hartree equation around some of its equilibria that are not spatially localized, such as the free Fermi gas at zero or positive temperature.

Etienne Sandier (Paris East Créteil): Bifurcating solitonic vortices in a strip

Abstract: I will report on results we have obtained with A.Aftalion and P.Gravejat which aim at giving a mathematical grounding of recent numerical simulations and experiments on Bose Einstein condensates. The experimental setup is that of an elongated condensate, which we model as an infinite strip. (joint work with Amandine Aftalion and Philippe Gravejat)

Benjamin Schlein (Uni Zurich): Dynamics of extended Fermi gases at high densities.

Abstract: We consider systems of N fermions initially trapped in a volume V, at high density. We show that, for initial data close to Slater determinants exhibiting an appropriate semiclassical structure, the solution of the many-body Schrödinger equation can be approximated by the solution of the nonlinear Hartree equation, up to errors that are small, for large density, uniformly in N and V. For fermions with relativistic dispersion, we prove that the approximation through the Hartree equation holds at the level of expectation of local observables. This is joint work with L. Fresta and M. Porta.

Ulrich Schollwöck (LMU Munich): TBA

Robert Seiringer (IST Austria): The effective mass problem for the polaron model

Abstract: The Fröhlich polaron and related models of quantum field theory have played a prominent role in mathematical physics over several decades. In this talk, we shall explain recent bounds on the quantum corrections to the (classical) Pekar approximation of the ground state energy of the Fröhlich polaron model in the strong coupling limit, and their consequence on the asymptotic behavior of the polaron's effective mass.

Jan Philip Solovej (Copenhagen): Is the periodic table of the elements periodic?

Abstract: Can the periodicity of the periodic table be explained mathematically? First of all we may ask what is the periodicity? The structure of the periodic table is phenomenologically described by the Aufbau principle or the Madelung rule. As we will explain, this simple rule will, however, fail if we consider atomic numbers Z tending to infinity (in a non-relativistic setting). On the other hand we may ask if there is nevertheless a perfect periodicity of atomic structure emerging in the limit of large Z. We will show that this is indeed the case in a simplified model of atoms This is joint work with August Bjerg, Søren Fournais, and Peter Hearnshaw.

Gigliola Staffilani (MIT): A curious phenomenon in wave turbulence theory

Abstract: In this talk we will use the periodic cubic nonlinear Schrödinger equation to present some estimates of the long time dynamics of the energy spectrum, a fundamental object in the study of wave turbulence theory. Going back to Bourgain, one possible way to conduct the analysis is to look at the growth of high Sobolev norms. It turns out that this growth is sensitive to the nature of the space periodicity of the system. I will present a combination of old and very recent results in this direction.

Luis Vega (Bilbao): The binormal flow and the evolution of viscous vortex filaments

Abstract: I'll present the so called Localized Induction Approximation that de- scribes the dynamics of a vortex filament according to the Binormal Curvature Flow (BF). I'll give a result about the desingularization of the Biot-Savart in- tegral proved with Marco A. Fontelos within the framework of Navier-Stokes equations. Some particular examples regarding BF obtained with Valeria Ban- ica will be also considered. These examples allow to connect BF with the so- called Riemann non-differentiable function and the Frisch-Parisi approach to turbulence.

Nicola Visciglia (Pisa): Construction of a sequence of invariant measures for the complex-valued modified-KdV equation

Abstract: we consider the complex valued mKdV, which is the second equation along the hierarchy associated with cubic NLS, and we construct a sequence of invariant measures associated with conservation laws of the equation. The main difficulty is that we work with complex valued functions, that makes more involved the problem compared to the real valued case considered previously by Zhidkov. This is a joint work with C. Kenig, A. Nahmod, N. Pavlovic, G.Staffilani.

Wilhelm Zwerger (TU Munich): Supersolids and self-bound droplets with ultracold gases

The talk will provide an introduction to the concept of supersolids which emerge from a uniform superfluid by the spontaneous formation of a density wave. We discuss the role of the Leggett bound for the superfluid fraction and the crucial distinction between a general mass-density wave and a commensurate solid. A concrete example is provided by dipolar gases, where the transition to a supersolid phase preempts the mean-field roton instability. Finally, it is shown that the long-range and anisotropic interactions in dipolar gases give rise to self-bound droplets which evolve into a needle-like shape in the thermodynamic limit.

Short talks:

Asbjørn Lauritsen (Paris Dauphine): Multi-band superconductors have enhanced critical temperatures

Abstract: Many superconductors are well-described by the Bardeen--Cooper--Schrieffer (BCS) theory of superconductivity. I will introduce this model and describe how the presence of multiple bands (think multiple flavours of charge carries) increases the critical temperature of the superconductor (the temperature below which the material is superconducting). This increase in the critical temperature is either (1) linear or (2) quadratic in the interband coupling if there are (1) multiple equally strongly superconducting bands or (2) a unique strongest superconducting band. Joint work with Joscha Henheik and Edwin Langmann.

Yahui Li (TU Munich): Highly-entangled stationary states from strong symmetries

Abstract: We find that the presence of strong non-Abelian conserved quantities can lead to highly entangled stationary states even for unital quantum channels. We derive exact expressions for the bipartite logarithmic negativity, Rényi negativities, and operator space entanglement for stationary states restricted to one symmetric subspace, with focus on the trivial subspace. We prove that these apply to open quantum evolutions whose commutants, characterizing all strongly conserved quantities, correspond to either the universal enveloping algebra of a Lie algebra or to the Read-Saleur commutants. The latter provides an example of quantum fragmentation, whose dimension is exponentially large in system size. We find a general upper bound for all these quantities given by the logarithm of the dimension of the commutant on the smaller bipartition of the chain. As Abelian examples, we show that strong U(1) symmetries and classical fragmentation lead to separable stationary states in any symmetric subspace. In contrast, for non Abelian SU(N) symmetries, both logarithmic and Rényi negativities scale logarithmically with system size. Finally, we prove that while Rényi negativities with $N>2$ scale logarithmically with system size, the logarithmic negativity (as well as generalized Rényi negativities with $N<2$) exhibits a volume law scaling for the Read-Saleur commutants. Our derivations rely on the commutant possessing a Hopf algebra structure in the limit of infinitely large systems, and hence also apply to finite groups and quantum groups.

Mirco Piccinini (Pisa): Harnack inequalities for kinetic integral equations

Abstract: We investigate local properties of weak solutions to a wide class of nonlocal equations arising in kinetic theory. Here, the diffusion term in velocity is an integro-differential operator, having nonnegative kernel of fractional order with merely measurable coefficients. Under sufficient integrability along the transport variables on the nonlocal tail, in turn based on the sharp gain integrability achievable via the fundamental solution and in clear accordance with the recent counterexample constructed in Kassmann \& Weidner (Adv. Math. 459, 2024), we provide an explicit local boundedness estimate and also show how to deduce from this result an Harnack inequality.

Stefano Rossi (Zurich): Asymptotic behavior of solutions to Vlasov-type equations

Abstract: In this talk, I will present some results on the asymptotic behavior of solutions to Vlasov-type equations. These nonlinear Liouville equations describe the evolution of a particle distribution function under the influence of a self-consistent force field.
The mathematical and physical properties of these systems vary between confined and unconfined physical spaces, exhibiting mixing or dispersive behavior, respectively. I will primarily focus on the unconfined case, examining how the asymptotic evolution of solutions can be free or include nonlinear corrections, depending on the dimension of the Euclidean space and the interaction type. Time permitting, I will discuss a recent result, with M. Iacobelli and K. Widmayer, on the stability around vacuum in low dimensions for the screened Vlasov-Poisson equation.

Jack Thomas (Orsay): Screening in the reduced Hartree-Fock model

Abstract: Placing a point charge Q at the origin in a vacuum produces a long-range Coulomb potential $\frac{Q}{\epsilon_0 r}$ with r being the distance to the charge and ϵ_0 the dielectric permittivity of the vacuum. However, in a material, the material reorganises itself; electrons move towards positive charges (or away from negative charges) and the total potential (including the Coulomb interactions, and the response from the electrons) is screened. Metals and insulators display fundamentally different screening behaviours. In metals at finite temperature, electrons are mobile and are able to move long distances to fully screen the defect; a simple empirical model being given by the Yukawa potential $\frac{Q}{r} e^{-k r}$. On the other hand, electrons in insulators are tightly bound to the nuclei and are thus unable to move too far from their periodic arrangement. A simple model for the total potential is given by $\frac{Q}{\epsilon r}$ for some $\epsilon > \epsilon_0$; one observes partial screening. This heuristic description has been rigorously proved for the reduced Hartree-Fock model at finite temperature, and for insulators at zero temperature. In metals at zero temperature, the sharp cut-off in the Fermi—Dirac distribution at the Fermi surface (the surface in reciprocal space separating occupied and unoccupied electron states) causes singularities in Fourier space. As a result, one expects to observe Friedel oscillations; the total potential decays with an algebraic rate with oscillatory tails. The rate of decay and frequency of oscillation depends on the Fermi

surface, and thus on the metal, in a non-trivial way. We will discuss ongoing research in this direction. Joint work with Eric Cances and Antoine Levitt.

Aleksis Vuoksenmaa (Helsinki): Chaos via joint cumulants -- the case of Kac's Stochastic model

Abstract: In this talk, we outline how chaos -- in the sense of near independence of the relevant random variables -- can be studied via the smallness of joint cumulants. Focusing on Kac's Stochastic model for velocity exchange of N particles, we show how to quantify the notion of chaos in terms of weighted norms on the space of joint cumulants of the energy variables. An analysis on how time-evolution of the joint cumulants in these spaces demonstrate how chaos is generated or propagated in the system. Known spectral gap results imply that typical initial densities converge to uniform distribution on the constant energy sphere at a time which has order of N expected collisions. We prove that the finite order cumulants converge to their small stationary values much faster, already at a time scale of order 1 or log(N) collisions. The proof relies on stability analysis of the closed, nonlinear hierarchy of energy cumulants around the fixed point formed by their values in the stationary spherical distribution. This talk is based on joint work with Jani Lukkarinen.

Posters:

Davide Desio (IST Austria): The Bosonization of Weakly Interacting Fermions

Abstract: A major problem in condensed matter is the derivation of effective theories to describe electron gases as a model for conducting materials. The Hartree-Fock theory is a well established approximation for fermionic systems, it is widely used to describe metals and atomic nuclei. Although the Hartree-Fock theory has been proved to be successful in the mean field regime hence for high density and weakly interacting fermionic systems, it fails in the description of the conductivity properties and specific heat. It is therefore important to go beyond the Hartree-Fock approximation and develop a deep understanding of the corrections to the Hartree-Fock approximation due to non trivial quantum correlations. In the 1950s, Bohm and Pines proposed the random phase approximation (RPA) as an effective theory to study the properties of high-density electron gases moving in a background of homogeneous positive charge called jellium. The random phase approximation can be interpreted as collective oscillations described by an effective bosonic hamiltonian whose ground state represents the leading order correction to the correlation energy. The justification of the RPA has been a major task in condensed matter and, re- cently, it has been proved rigorously for the mean field regime in the case of compactly supported, long range and Coulomb interactions. We revisit the bosonization approach to the RPA corrections in the same spirit of the transformations implemented to study the excitation spectrum of weakly interacting bosons. This approach allows to produce more precise estimates on the RPA corrections for the case of a compactly supported potential with respect to the already existing results. This is a joint work with Niels Benedikter from the University of Milan.

Florent Fougères (Paris): Deriving the linear Rayleigh–Boltzmann equation: adaptive pruning and long-time results

Abstract: For dilute gases, the derivation of the Boltzmann equation (1872) from the Newton microscopic equations has been mathematically conducted in 1975 by Oscar Lanford; and in the 2010s Isabelle Gallagher, Laure Saint-Raymond and Benjamin Texier gave precise quantitative estimates on the convergence of the empirical measure to the solution to this equation, yet only for very short times. In a linear context - close to equilibrium - this work has allowed long-time results, later on used to derive hydrodynamic limits - as suggested by the Hilbert's sixth problem. In this presentation we show a more precise approach to prove far faster convergence rates in this linear case, based on a more physics-inspired time sampling, so as to give these results more physical meaning.

Antoine Gagnebin (Zurich): Landau damping and final data problem for Vlasov-type equations.

Abstract: I will present Vlasov-type equations on the torus T^d, which are partial differential equations (PDEs) used to model confined plasma physics, particularly in scenarios where collisions between particles and external magnetic effects are neglected. The focus will be on a class of equilibria for these equations and the effects of perturbations around these equilibria. While the long-term behaviour of solutions given initial data (the Cauchy problem) is well understood through the phenomenon known as Landau damping, I will present a new result concerning the final data problem. Specifically, instead of starting with an initial condition for the PDE, we consider the problem where a final state is given.

Borbala Gerhat (IST Austria): Schur complement dominant operator matrices

Abstract: Due to the matrix structure, spectral problems arising from systems of coupled linear partial differential equations can be intrinsically challenging. A successful approach motivated already on the level of scalar matrices is to relate the operator matrix (to be implemented in a product Hilbert space) to one of its Schur complements. In the unbounded non-selfadjoint operator set- ting, we introduce a robust abstract method to rigorously establish this connection and thereby extend previous approaches which use relative boundedness within the matrix entries. The cor- nerstones of our method are a Schur complement which is dominant in a suitable sense with respect to the entries, as well as a distributional approach to implementing the matrix opera- tions. This allows to pass from a well-behaved representation of the Schur complement (e.g. by its sesquilinear form) to a well-behaved representation of the operator matrix, and to estab- lish an equivalence between the (point and essential) spectra of matrix and Schur complement. Applications range for instance from damped wave equations to Dirac operators with strong damping or potential.

Emanuela Giacomelli (LMU Munich): The Huang-Yang formula for the low-density Fermi gas

Abstract: We investigate the ground state energy of spin-1/2 fermions with repulsive short-range interactions at low density ρ. Our result validates the Huang-Yang conjecture by deriving the first three terms in the asymptotic low-density expansion, including the correction of order $\rho^{7/3}$.

Jakob Oldenburg (Zurich): Quantum Fluctuations of Many-Body Dynamics around the Gross-Pitaevskii Equation

Abstract: We consider the evolution of a gas of N bosons in the three-dimensional Gross-Pitaevskii regime (in which particles are initially trapped in a volume of order one and interact through a repulsive potential with scattering length of the order 1/N). We construct a quasi-free approximation of the many-body dynamics, whose distance to the solution of the Schrödinger equation converges to zero, as N goes to infinity, in the L^2-norm. To achieve this goal, we let the Bose-Einstein condensate evolve according to a time-dependent Gross-Pitaevskii equation. After factoring out the microscopic correlation structure, the evolution of the orthogonal excitations of the condensate is governed instead by a Bogoliubov dynamics, with a time-dependent generator quadratic in creation and annihilation operators. As an application, we show a central limit theorem for fluctuations of bounded observables around their expectation with respect to the Gross-Pitaevskii dynamics. This is joint work with Cristina Caraci and Benjamin Schlein.

Florian Haberberger (LMU Munich): The free energy of dilute bose gases at low temperatures.

Abstract: We consider a system of bosons in the thermodynamic limit. For small densities ρ, Lee-Huang-Yang (1957) conjectured an expansion of the free energy of the system including a leading term of order ρ^2 and the LHY correction term of order $\rho^{5/3}$. Their work also includes a prediction about the low lying excitation spectrum. In a series of two papers we were able to verify the energy expansion at positive temperature, which in particular supports the excitation spectrum hypothesis. This is joint work with: C.Hainzl, P.T.Nam, B.Schlein, R.Seiringer, A.Triay.

Alessandro Olgiati (Milano): Ground state energy upper bounds for dilute Bose gases with hard-core interactions

Abstract: Recent advances in the understanding of the ground state properties of dilute Bose gases have led to the rigorous proof of the celebrated Lee-Huang-Yang (LHY) asymptotic formula. Despite such progress, one relevant case has proven elusive: even though all relevant quantities are well defined for a large class of repulsive non-locally integrable interaction potentials, those potentials are not covered by existing proofs of the LHY formula as an energy upper bound. The poster reviews recent results on the existence of trial states that match LHY-type formulae for the hard-core potential. The optimal asymptotics was recovered in the Gross-Pitaevskii regime, while the optimal order of magnitude (with non-sharp proportionality constant) was reached in the thermodynamic limit. Based on joint works with G. Basti, S. Cenatiempo, A. Giuliani, G. Pasqualetti, and B. Schlein.

Jonas Peteranderl (LMU Munich): Degenerate Stability of the Caffarelli-Kohn-Nirenberg Inequality along the Felli-Schneider Curve

Abstract: We show that the Caffarelli-Kohn-Nirenberg inequality holds with a remainder term that is quartic in the distance to the set of optimizers for the full parameter range of the Felli–Schneider curve. The fourth power is best possible. This is due to the presence of non-trivial zero modes of the Hessian of the deficit functional along the FS-curve. Following an iterated Bianchi–Egnell strategy, the heart of our proof is verifying a ‘secondary non-degeneracy condition’. Our result completes the stability analysis for the CKN-inequality to leading order started by Wei and Wu. Moreover, it is the first instance of degenerate stability for non-constant optimizers and for a non-compact domain. This is joint work with Rupert Frank.

Lorenzo Pigozzi (IST Austria): Longitudinal conductivity at Hall topological phase transitions

Abstract: We investigate topological phase transitions in translation-invariant quantum Hall systems, establishing an explicit formula for the longitudinal conductivity It is known that, generically, the phase transition between different Hall phases takes place in correspondence with conical intersections of the Bloch bands at the Fermi energy. We consider a family of one-body tight-binding Hamiltonians whose main assumption is the conical structure of the two eigenvalues near the Fermi energy. The longitudinal conductivity turns out to be determined only by the number of conical intersections and the geometry of the cones. In particular, this formula gives the quantized value known for the graphene and the Haldane model at critical regime. This is a joint work with G. Marcelli and M. Porta.

Sakari Pirnes (Helsinki): Well-posedness and parameter dependence of the truncated cumulant hierarchy for Bosons and NLS on finite lattices

Abstract: We study the cumulant hierarchy of Bosons and NLS on a lattice, truncated by dropping the higher order cumulants in the usual manner which would lead to wave kinetic equations. The harmonic part is given by nearest neighbor hopping, with onsite interaction potential of coupling strength $\lambda>0$. We consider the well-posedness of the resulting evolution equation up to finite kinetic times on a finite but large enough lattice. We show that the solutions are not sensitive to how the energy conservation delta function is approximated. The error is bounded by a positive power of λ, uniformly in the lattice size. These results are based on a work in progress with Jani Lukkarinen and Aleksis Vuoksenmaa.

Larry Read (LMU Munich): Concentration of eigenfunctions in two physical models

Abstract: This poster presents two phenomena: the concentration of low-energy states in quantum wires subject to a constant electric field, and the concentration of high-frequency

acoustic modes in gas giants. Low-energy states in quantum wires localise at the boundary in the direction of the electric field, as its strength increases. Similarly, in gas giants, high-frequency acoustic modes localise at the boundary. We study the density of these eigenfunctions and determine the rates of concentration in terms of the strength of the electric field for quantum wires, and the frequency for gas giants.

Tobias Ried (Georgia Tech): Regularity of optimal transport and applications

Abstract: We present a fully variational approach to the regularity theory of optimal transportation, following De Giorgi's strategy for the regularity theory of minimal surfaces. It is based on the approximation of the displacement by a harmonic gradient, and leads to a quantitative linearization result for the Monge-Ampère equation. This turns out to be related to branched microstructures that give rise to the complex patterns at the surface of type-I superconductors in the intermediate state. In a different direction, we consider the structure of multi-marginal optimal transport (MMOT) problems related to p-Wasserstein barycenters. As a first step towards a regularity theory for MMOT we prove that optimal plans are sparse.

François Visconti (LMU Munich): On stability of trapped two-dimensional Bose gases with attractive interactions

Abstract: In this poster we discuss questions and recent results on trapped two-dimensional Bose gases with attractive interactions. Such systems are particularly difficult to deal with because even the stability of the many-body system is not obvious, as shall be discussed in this poster. Moreover, the convergence of the ground state energy per particle to that of a non-linear Schrödinger (NLS) energy functional shall also be discussed. Based on joint work with Lukas Junge.