LMU LUDWIG-MAXIMILIANS-UNIVERSITÄT MÜNCHEN
MATHEMATISCHES INSTITUT

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Tutorial sheet 1
Algebra

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Exercise 1. Let G be a group with subgroups $H_1, H_2 \subseteq G$. Show that $H_1 \cup H_2$ is a subgroup of G if and only if $H_1 \subseteq H_2$ or $H_2 \subseteq H_1$.

Exercise 2. (a) Prove that the additive group $(\mathbb{R}, +)$ is isomorphic to the multiplicative group $(\mathbb{R}^+, \cdot)$. Does the same hold for $(\mathbb{Q}, +)$ and $(\mathbb{Q}^+, \cdot)$?

(b) Prove that $(\mathbb{R}^\times, \cdot)$ and $(\mathbb{R}, +)$ are not isomorphic as groups.

(c) Let $m, n \in \mathbb{Z}$ be coprime integers and let $f : \mathbb{Z}/m\mathbb{Z} \to \mathbb{Z}/n\mathbb{Z}$ be a group homomorphism. Prove that f is trivial, i.e. $f \equiv 0$.

(d) Describe all group homomorphisms $(\mathbb{Z}/4\mathbb{Z}, +) \to (\mathbb{Z}/6\mathbb{Z}, +)$.

Exercise 3. Let G be a group such that for all $g \in G$, $g^2 = 1$. Show that G is abelian.

Exercise 4. Let G be a finite abelian group. Show that

$$\prod_{g \in G} g^2 = 1.$$