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Repetition week 6
Algebra

Winter term 25/26

This repetition sheet is comparatively short, as the lecture covered the proofs of the three Sylow theorems as well as the full classification of the finite subgroups of SO(3), whose proof is relatively long.

Finite subgroups of SO(3)

Theorem. Let $G \subseteq SO(3)$ be a finite subgroup. Then:

1) Either G stabilizes some plane $H \subseteq \mathbb{R}^3$, and then

$$G \hookrightarrow O(H) \cong O(2).$$

In this case, G is a cyclic group of order n, i.e. $G \cong \mathbb{Z}/n\mathbb{Z}$, or a dihedral group D_n of order $2n$ (the symmetry group of a regular n-gon lying in H), where

$$D_n \;\cong\; \mathbb{Z}/n\mathbb{Z} \rtimes \mathbb{Z}/2\mathbb{Z}.$$

2) Or G stabilizes no plane, in which case G must be one of the following:

   a) $G \cong A_4$ (rotational symmetry group of the regular tetrahedron),
   b) $G \cong S_4$ (rotational symmetry group of the cube / octahedron),
   c) $G \cong A_5$ (rotational symmetry group of the icosahedron / dodecahedron).