Advanced Mathematical Quantum Mechanics – Homework 6

Mathematisches Institut der LMU – SS2009
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Exercise 6.1. Compute the following integral (occurring in the proof of the *relativistic Hardy inequality*):

$$\lim_{\varepsilon\to 0}\int_0^\infty \mathrm{d}t\,\frac{t(t-A)}{(t^2-A^2)^2+2\varepsilon(t^2+A^2)+\varepsilon^2} \qquad (A\geqslant 0)\,.$$

Exercise 6.2. Compute the following integral (occurring in the proof of the *localised relativistic kinetic energy inequality*):

$$\mathrm{p.v.}\int_0^1 \mathrm{d}t\,\frac{2t^2}{(t^2-A^2)^2}\Big(1-\frac{h(t)}{h(A)}\Big) \qquad (0<A<1)$$

where $h(t)=1/t+t$. In the case of a generic $h\in C^1(]0,1],\mathbb{R})$ that is *strictly decreasing* on $]0,1[$, what is the necessary condition on $h'(1)$ so that the integral above remains bounded when $A\to 1$?