AB Geometrie & Topologie
Dr. Christian Lange

Symplectic Geometry and Hamiltonian Dynamics

Lecture WS 2024/25

as of October 1, 2024

Symplectic structures naturally arise in classical mechanics, particularly Hamiltonian dynamical systems. For instance, Liouville's theorem about phase space volume invariance under the Hamiltonian flow is actually a consequence of the preservation of a natural symplectic form.

In a celebrated work from 1985 Gromov observed that symplectic maps (like the Hamiltonian flow) show rigidity phenomena that do not occur for merely volume-preserving maps. Gromov's *non-squeezing theorem*, which can be interpreted as a classical analogue of the quantum mechanical uncertainty principle, laid the foundation for a new mathematical field, symplectic topology, the study of global phenomena in symplectic geometry.

The goal of the lecture is to give an introduction to symplectic geometry with an emphasis on Hamiltonian dynamics. In particular, we will discuss existence results for periodic orbits of Hamiltonian systems and a related proof of Gromov's nonsqueezing theorem via methods from the calculus of variation.

For master students of mathematics and physics.

Prerequisites: Linear Algebra I+II and Analysis I-III (calculus of one and several real variables, measure and integration), basics on abstract manifolds, differential forms, ODE, Hilbert spaces.

Time: Wed+Thu 8:30 - 10 am.

References:

A. C. da Silva, *Lectures on Symplectic Geometry*, Springer 2008
H. Geiges, *An introduction to contact topology*, Cambridge University Press 2005
H. Hofer, E. Zehnder, *Symplectic invariants and Hamiltonian dynamics*, Birkhäuser 1994
D. McDuff, D. Salamon, *Introduction to symplectic topology*, Oxford University Press 2017
E. Zehnder, *Lectures on Dynamical systems*, EMS 2010

To participate please register in Moodle (enrolment key "Gromov").