LUDWIG-
MAXIMILIANS-
UNIVERSITÄT
MÜNCHEN

MATHEMATISCHES INSTITUT

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Lecture: Riemann surfaces

Exercise sheet 9

Exercise 1. Let X be a disk of radius $0 < R \leq \infty$ and let $\mathcal{H}$ denote the sheaf of harmonic functions. Show that $H^1(X, \mathcal{H}) = 0$.

Exercise 2. Let $g \in \mathcal{E}(\mathbb{C})$ be a smooth function with compact support. Prove that there is a function $f \in \mathcal{E}(\mathbb{C})$ with compact support which satisfies

$$\frac{\partial f}{\partial \bar{z}} = g$$

if and only if

$$\int_{\mathbb{C}} z^n g(z) \mathrm{d}z \wedge \mathrm{d}\bar{z} = 0 \qquad \forall n \in \mathbb{N}_0$$

Hint: If $|\zeta| > |z|$ then one may turn

$$\frac{1}{z - \zeta}$$

into an expression involving the geometric series.

Exercise 3. Show that the following are short exact sequences of sheaves on a Riemann surface:

(a)

$$1 \longrightarrow \mathbb{C}^* \longrightarrow \mathcal{O}^* \xrightarrow{\mathrm{d}\log} \Omega \longrightarrow 0$$

where $\mathrm{d}\log : \mathcal{O}^* \to \Omega$ denotes the homomorphism $f \mapsto \mathrm{d}f/f$.

(b)

$$0 \longrightarrow \mathbb{C} \longrightarrow \mathcal{M} \xrightarrow{\mathrm{d}} \mathcal{Q} \longrightarrow 0$$

where $\mathcal{M}$ denotes the sheaf of meromorphic functions and $\mathcal{Q}$ the sheaf of meromorphic one-forms with residue 0 at every pole.

Please hand in your solutions at the start of the exercise class on **Monday, July 11, 2017**.