Dr. Mark Hamilton
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Lecture: Riemann surfaces

Exercise sheet 6

Exercise 1. Let $\mathcal{F}$ be a presheaf which satisfies the locality axiom and $|\mathcal{F}|$ its associated topological space (also called the *étalé space*). Prove that if $|\mathcal{F}|$ is Hausdorff, then $\mathcal{F}$ satisfies the identity theorem.

Exercise 2. Show that the coboundary map δ on q-cochains satisfies $\delta^2 = 0$, $\forall q \in \mathbb{N}$.

Exercise 3. Show that on any smooth manifold the first cohomology group with coefficients in the sheaf of smooth one-forms, $H^1(X, \mathcal{E}^1)$, vanishes. Observe (no proof needed) that, if the sheaf of sections of a given vector bundle admits partitions of unity, then an analogous proof yields the same result.

Exercise 4. Let $P = \{p_1, \ldots, p_n\}$ be a finite set of points in $\mathbb{C}$ and set $X = \mathbb{C} \setminus P$. Prove that $H^1(X, \mathbb{Z}) \cong \mathbb{Z}^n$ by using a Leray covering consisting of two open subsets.

Please hand in your solutions at the start of the exercise class on **Monday, June 19, 2017**.