LUDWIG-MAXIMILIANS-UNIVERSITÄT MÜNCHEN
MATHEMATISCHES INSTITUT

Dr. Mark Hamilton
Danu Thung

Summer semester 2017

Lecture: Riemann surfaces

Exercise sheet 4

Exercise 1. Consider the Weierstrass $\wp_\Gamma$-function from exercise sheet 3 as a branched covering $\mathbb{C}/\Gamma \to \mathbb{CP}^1$. Show that it is two-sheeted, and use the fact that its derivative is an odd function to show that it has exactly four branch points.

Exercise 2. Show that every holomorphic map from the sphere $\mathbb{CP}^1$ to a torus $\mathbb{C}/\Gamma$ is constant, i.e. there are no branched coverings $f\colon \mathbb{CP}^1 \to \mathbb{C}/\Gamma$.

Exercise 3. Let $f\colon \mathbb{C}/\Gamma \to \mathbb{C}/\Gamma'$ be a branched covering with $f([0]) = [0]$.

(a) Show that there exists some $\alpha \in \mathbb{C}^*$ such that $\alpha\Gamma \subset \Gamma'$ and $f([z]) = [\alpha z]$ for every $[z] \in \mathbb{C}/\Gamma$.

(b) Show that f is in fact unbranched, and the number of sheets equals the index $[\Gamma' : \alpha\Gamma]$.

(c) Construct n-sheeted holomorphic coverings of the standard torus $\mathbb{C}/(\mathbb{Z} + \mathbb{Z}i)$ to itself, for any $n \in \mathbb{N}$ such that there exist $k, l \in \mathbb{N}_0$ which satisfy $n = k^2 + l^2$.

Please hand in your solutions at the start of the exercise class on **Monday, May 29, 2017**.