

ADVANCED ANALYSIS – WiSe 2019/20

Exercise sheet 1

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Exercise 1. [20 points]

Consider the following sequences in $L^p(\mathbb{R})$, with $1 < p < \infty$:

1. $f_k(x) := \begin{cases} \sin kx & \text{for } 0 \leq x \leq 1, \\ 0 & \text{otherwise.} \end{cases}$
2. $g_k(x) := k^{\frac{1}{p}} g(kx)$, where g is any fixed function in $L^p(\mathbb{R})$.
3. $h_k(x) := g(x+k)$ for some fixed function g in $L^p(\mathbb{R})$.

Prove that f_k, g_k, h_k converge weakly to 0 but do not converge strongly to 0 (or to anything else).

Exercise 2. [20 points]

Let $|t| \leq 1$, prove that for any f, g both these inequalities hold true:

$$|f|^p - |f - g|^p \leq \frac{1}{t} \{|f + tg|^p - |f|^p\} \leq |f + g|^p - |f|^p.$$