LMU LUDWIG-MAXIMILIANS-UNIVERSITÄT MÜNCHEN

MATHEMATISCHES INSTITUT

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Functional Analysis

Excercise Sheet 2

Remaining solutions

Problem 1 (Closure, interior, boundary w.r.t. complement, inclusion, union, and intersection)**.** Let $(X, \mathcal{T})$ be a topological space and $E, F \subseteq X$.

f) Prove that $\partial(E \cup F) \subset \partial E \cup \partial F$ and find an example of strict inclusion.

Proof. f) Let $x \in \partial(E \cup F)$. Assume $x \notin \partial E \cup \partial F$. Then there exists nbhds U and V of x s.t. $U \cap E = \emptyset \vee U \cap (X \setminus E) = \emptyset$ and $V \cap E = \emptyset \vee V \cap (X \setminus E) = \emptyset$. But then $W := U \cap V$ is a nbhd of x with $W \subseteq U$, $W \subseteq V$. Thus $W \cap E = \emptyset \vee W \cap (X \setminus E) = \emptyset$ and $W \cap E = \emptyset \vee W \cap (X \setminus E) = \emptyset$. If $W \cap E = \emptyset$ and $W \cap F = \emptyset$ then $W \cap (E \cup F) = (W \cap E) \cup (W \cap F) = \emptyset$ in contradiction to $x \in \partial(E \cup F)$. Otherwise (at least) $W \cap (X \setminus E) = \emptyset$ or $W \cap (X \setminus F) = \emptyset$ and thus $W \cap (X \setminus (E \cup F)) = W \cap (X \setminus E) \cap (X \setminus F) = \emptyset$ in contradiction to $x \in \partial(E \cup F)$.

An example for strict inclusion in $(\mathbb{R}, \mathcal{T}_{Eucl})$ is $E := [0, 1]$ and $F := [1, 2]$. Then $\partial E = \{0, 1\}$ and $\partial F = \{1, 2\}$ and thus $\partial E \cup \partial F = \{0, 1, 2\}$. On the other side $\partial(E \cup F) = \{0, 1\}$.

□

Deadline: May 2, 2016*, for details see http://www.math.lmu.de/˜gottwald/16FA/.*