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Functional Analysis
Tutorial 12

Problem 1. Let X be a normed space. Prove:

(i) Weakly bounded sets in X are bounded.

(ii) Weak Cauchy sequences in X are bounded.

Problem 2. Let X be a normed space. Prove:

(i) For all $x \in X$ there exists $F \in X'$ such that $F(x) = \|x\|$ and $\|F\| = 1$.
[*Hint:* Theorem 4.16.]

(ii) X' separates points in X, i.e. for all $x, y \in X$ with $x \neq y$, there exists $F \in X'$ such that $F(x) \neq F(y)$.

(iii) Weak limits are unique.

Problem 3. Let X be a normed space, let $n \in \mathbb{N}$, let $x_1, \dots, x_n \in X$ be linearly independent, and let $\alpha_1, \dots, \alpha_n \in \mathbb{C}$. Show that there exists $F \in X'$ such that

$$F(x_j) = \alpha_j \qquad \forall\, j \in \mathbb{N} \text{ with } 1 \leqslant j \leqslant n.$$