LUDWIG-MAXIMILIANS-UNIVERSITÄT MÜNCHEN
MATHEMATISCHES INSTITUT

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Functional Analysis
Tutorial 1

Problem 1 (Trivial topology). Let X be a non-empty set and let $\mathcal{T}_0 := \{\emptyset, X\}$ be the trivial (or *indiscrete*) topology on X.

(*i*) Find the closure $\overline{A} \subset X$ with respect to $\mathcal{T}_0$ of a subset $A \subset X$.

(*ii*) Prove that for any topological space $(Y, \mathcal{T})$, any map $f : (Y, \mathcal{T}) \to (X, \mathcal{T}_0)$ is continuous.

(*iii*) Let $\mathcal{T}$ be a topology on X that is *not* trivial. Decide whether the identity maps $id_1 : (X, \mathcal{T}_0) \to (X, \mathcal{T})$ and $id_2 : (X, \mathcal{T}) \to (X, \mathcal{T}_0)$ are continuous.

Problem 2 (Discrete topology). Let X be a non-empty set and let $\mathcal{T}_d := \mathcal{P}(X) = 2^X$ be the discrete topology on X, i.e. the topology consisting of all subsets of X.

(*i*) Prove that $(X, \mathcal{T}_d)$ is a Hausdorff space.

(*ii*) Find the closure $\overline{A} \subset X$ with respect to $\mathcal{T}_d$ of a subset $A \subset X$.

(*iii*) Prove that for any topological space $(Y, \mathcal{T})$, any map $f : (X, \mathcal{T}_d) \to (Y, \mathcal{T})$ is continuous.

(*iv*) Prove that if $\mathcal{T}$ is a topology on X that contains all singletons, i.e. $\{x\} \in \mathcal{T}$ for all $x \in X$, then $\mathcal{T} = \mathcal{T}_d$.

(*v*) Let $\mathcal{T}$ be a topology on X that is *not* discrete. Decide whether the identity maps $id_1 : (X, \mathcal{T}_d) \to (X, \mathcal{T})$ and $id_2 : (X, \mathcal{T}) \to (X, \mathcal{T}_d)$ are continuous.

Problem 3 (Relative topology). Let $(X, \mathcal{T})$ be a topological space, let $Y \subset X$ be a subset, and let $\mathcal{T}_Y$ denote the relative topology on Y induced by $\mathcal{T}$.

(*i*) Prove that Y is open in $(X, \mathcal{T})$, i.e. $Y \in \mathcal{T}$, if and only if all open sets in $(Y, \mathcal{T}_Y)$ are open in $(X, \mathcal{T})$, i.e. $\mathcal{T}_Y \subset \mathcal{T}$.

(*ii*) Prove the statement in (*i*) where *open* is replaced by *closed*.

(*iii*) Let $Z \subset X$ be another subset. Prove that if $U \subset Y$ is open in $(Y, \mathcal{T}_Y)$, then $U \cap Z$ is open in $(Y \cap Z, \mathcal{T}_{Y \cap Z})$.

(*iv*) Let $Z \subset X$ and assume that $U \subset Y \cap Z$ is open in $(Y, \mathcal{T}_Y)$ and $(Z, \mathcal{T}_Z)$. Prove that U is open in $(Y \cup Z, \mathcal{T}_{Y \cup Z})$, i.e. $\mathcal{T}_Y \cap \mathcal{T}_Z \subset \mathcal{T}_{Y \cup Z}$.

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