

Titles & Abstracts

Wednesday, July 5

Marcello Porta

Dynamics of high density Fermi gases

I will discuss the quantum evolution of many-body Fermi gases in three dimensions, in arbitrarily large domains. I will focus on a high-density/semiclassical scaling, and I will consider a class of initial data describing zero temperature states with a suitable semiclassical structure. We prove that, as the density goes to infinity, the many-body evolution of the reduced one-particle density matrix converges to the solution of the time-dependent Hartree equation, for short macroscopic times. In the case of a relativistic dispersion, the result extends to all macroscopic times. With respect to previous work, the rate of convergence does not depend on the total number of particles, but only on the density of particles: in particular, our result allows us to study the dynamics of extensive many-body Fermi gases. Joint work with Luca Fresta and Benjamin Schlein.

Robin Reuvers

Ground state energy of dilute Bose and Fermi gases in 1D

I will discuss dilute, repulsive Bose and Fermi gases in one dimension. For the case without spin, I will mention some aspects of the ground state energy expansion obtained with Johannes Agerkov and Jan Philip Solovej. For the case with spin, I will explain how to think about the ground state, and I will discuss the upper bound obtained with the same co-authors.

Morris Brooks

The Fröhlich Polaron at strong coupling

In this talk we will discuss recent results of Robert Seiringer and myself, concerning the ground state energy and the energy-momentum relation of the Fröhlich polaron, which is a model describing the interactions between a charged particle and a polarized medium. We especially verify a conjecture by Landau and Pekar from 1948, claiming that the energy-momentum relation asymptotically coincides with the one of a free particle having an increased mass $M = \alpha^4 m$, where m is an explicit constant, in the regime of large couplings α between the particle and the medium, and suitably small momenta.

Marie Fialova

Asymptotics of weakly coupled eigenvalues of the Pauli operator:

We are interested in the magnetic Pauli operator perturbed by a potential ϵV , where ϵ is a small parameter. In particular, we want to find the asymptotics of the eigenvalues as ϵ tends to zero. We consider the Aharonov-Bohm magnetic field and show that both components of the Pauli operator are critical in the sense of not satisfying a Hardy-type inequality. Consequently the small potential perturbation results in emergence of two eigenvalues compared to the usual result of only one eigenvalue for non-singular magnetic fields. This is a joint project in progress with David Krejcirik.

Thursday, July 6

Julian Fischer

A rigorous approach to the Dean-Kawasaki equation of fluctuating hydrodynamics

Fluctuating hydrodynamics provides a framework for approximating density fluctuations in interacting particle systems by suitable SPDEs. The Dean-Kawasaki equation - a strongly singular SPDE - is perhaps the most basic equation of fluctuating hydrodynamics; it has been proposed in the physics literature to describe the fluctuations of the density of N diffusing weakly interacting particles in the regime of large particle numbers N . The strongly singular nature of the Dean-Kawasaki equation presents a substantial challenge for both its analysis and its rigorous mathematical justification: Besides being non-renormalizable by approaches like regularity structures, it has recently been shown to not even admit nontrivial martingale solutions.

In this talk, we give an overview of recent quantitative results for the justification of fluctuating hydrodynamics models. In particular, we give an interpretation of the Dean-Kawasaki equation as a "recipe" for accurate and efficient numerical simulations of the density fluctuations for weakly interacting diffusing particles, allowing for an error that is of arbitrarily high order in the inverse particle number.

Based on joint works with Federico Cornalba, Jonas Ingmanns, and Claudia Raithel-

Jinyeop Lee

A mixed-norm estimate of two-particle reduced density matrix of many-body Schrödinger dynamics for deriving Vlasov equation

We re-examine the combined semi-classical and mean-field limit in the N -body fermionic Schrödinger equation with pure state initial data using the Husimi measure framework. The

Husimi measure equation involves three residue types: kinetic, semiclassical, and mean-field. The main result of this paper is to provide better estimates for the kinetic and mean-field residue than those in Chen et al. (J Stat Phys 182(2):1–41, <http://arxiv.org/abs/1910.09892v4>, 2021). Especially, the estimate for the mean-field residue is shown to be smaller than the semiclassical residue by a mixed-norm estimate of the two-particle reduced density matrix factorization. Our analysis also updates the oscillation estimate parts in the residual term estimates appeared in Chen et al. (J Stat Phys 182(2):1–41, <http://arxiv.org/abs/1910.09892v4>, 2021). This is a joint work with Li Chen, Yue Li and Matthew Liew.

Martin Ravn Christiansen

Emergent Quasi-Bosonicity in Interacting Fermi Gases

We consider the correlation energy of a Fermi gas on a torus as the particle number N goes to infinity, with the interaction potential scaled by a factor proportional to $N^{-1/3}$.

In the second-quantized picture, the Hamiltonian of such a system can be written in the form of a quadratic Hamiltonian with respect to certain "quasi-bosonic" operators. By applying the theory of bosonic Bogolubov transformations, this Hamiltonian can be approximately diagonalized to yield a "bosonic contribution" to the correlation energy.

In this talk we will see how this quasi-bosonic behavior emerges, and how to efficiently carry out such a diagonalization procedure in the non-exact setting.

Giulia Basti

Upper bounds on the ground state energy of dilute hard core bosons

In this talk we discuss some recent estimates on the energy of dilute gases of hard core bosons. We present an upper bound for hard core bosons in the thermodynamic limit resolving the ground state energy up to an error of the order of the so-called Lee-Huang-Yang correction. We also discuss the Gross-Pitaevskii regime, in which N hard spheres with radius of order $1/N$ move on the unit torus; in this setting, we show an upper bound for the ground state energy up to an error vanishing as N diverges.

Based on joint works with S. Cenatiempo, A. Giuliani, A. Olgiati, G. Pasqualetti, B. Schlein.

Toan Nguyen

Plasma oscillations

Charged particles in a non-equilibrium state can experience complex behavior at large times, including trapped trajectories, oscillations, phase mixing, and Landau damping.

This talk is to identify the survival threshold for oscillations of an electron modeled by Hartree or Vlasov equations.

Lea Boßmann

Focusing NLS and Bogoliubov correction for dilute Bose gases in the instability regime

We consider the dynamics of a $2d$ Bose gas with singular attractive interactions in the instability regime, where the corresponding focusing nonlinear Schrödinger equation (NLS) has a blow-up. We show that the evolution of the condensate is effectively described by this NLS for all times before the blow-up. Moreover, we prove the validity of the Bogoliubov approximation for the dynamics of the fluctuations, resulting in a norm approximation of the many-body dynamics. This is joint work with Charlotte Dietze and Phan Thành Nam.

Asbjørn Bækgaard Lauritsen

Dilute Fermi gases: Upper bounds via cluster expansion

Recently the study of dilute quantum gases have received much interest, in particular regarding their ground state energies. I will present recent work on two such problems: that of a spin-polarized Fermi gas and a spin-1/2 Fermi gas. In the spin-polarized setting the ground state energy is bounded from above by the kinetic energy plus a term of order $a^3 \rho^{8/3}$ with a the p -wave scattering length of the repulsive interaction. In the spin-1/2 setting the correction is of order $a_s \rho_\uparrow \rho_\downarrow$ with a_s the s -wave scattering length. One of the main ingredients in the proofs is a rigorous version of a formal cluster expansion of Gaudin, Gillespie and Ripka (Nucl. Phys. A, 176.2 (1971), pp. 237-260). I will discuss this expansion and the analysis of its absolute convergence.

Joint work with Robert Seiringer. Based on arXiv:2301.04894 and 2301.08005.

Luca Fresta

Fermionic Stochastic Quantization

Stochastic quantization is the study of measures via the push-forward from Gaussian measures. The goal of my talk is to survey some recent results on the application of stochastic quantization in the context of Grassmann measures, e.g., fermionic euclidean quantum field theories, which are an important example of non-commutative measures. Based on joint works with F. De Vecchi, M. Gordina and M. Gubinelli.

Friday, July 7

Jacky Chong

Norm approximation of weakly interacting bosons

In this talk, we study the mean-field dynamics of a many-body bosonic system and obtain a quantitative bound on the error between the exact and effective dynamics. More precisely, we consider a system of N interacting bosons where the particles experience a short range two-body interaction given by $N^{-1}v_N(x) = N^{3\beta-1}v(N\beta x)$ where $0 < \beta < 1$ and v is a non-negative spherically symmetric function. Our new main result is to obtain a global-in-time Fock space norm estimate on the error between the many-body dynamics and mean-field dynamics with sublinear growth in time. Moreover, our extension allows for a more general set of initial data that includes coherent states. The talk is based on a joint work with M. Grillakis, M. Machedon, X. Huang, and Z. Zhao.

Charlotte Dietze

Spectral estimates for Schrödinger operators with Neumann boundary conditions on Hölder domains

We prove a universal bound for the number of negative eigenvalues of Schrödinger operators with Neumann boundary conditions on bounded Hölder domains, under suitable assumptions on the Hölder exponent and the external potential. Our bound yields the same semiclassical behaviour as the Weyl asymptotics for smooth domains. We also discuss different cases where Weyl's law holds and fails.

Ngoc-Nhi Nguyen

Weyl laws for interacting particles

The known Weyl laws of Schrödinger operators $-\hbar^2\Delta + V$ provide asymptotics the ground state density of systems of several non-interacting fermions submitted to an external potential V . We will discuss the corresponding version in the case of interactions between these particles.

Torben Krüger

Merging singularities in two-dimensional Coulomb gases

The two-dimensional one-component plasma is a particle system in the plane with long-range logarithmic interactions. At a specific temperature the system is equivalent to the

eigenvalue ensemble of a normal random matrix model. In equilibrium the particles form distinct droplets when placed in an external potential. Using the Riemann-Hilbert approach we determine the local statistical behaviour of the particles at the point where two droplets merge and observe an anisotropic scaling behaviour with particles being much further apart in the direction of merging than the perpendicular direction.

This is joint work with Meng Yang and Seung-Yeop Lee.
