Mathematisches Institut
der Universität München
Prof. Otto Forster

WS 2012/13
December 7, 2012

Riemann Surfaces

Problem sheet #8

Problem 29

Let $X := \mathbb{C}/\Lambda$ be a torus and $\Omega(X)$ be the vector space of all holomorphic 1-forms on X. Prove that $\dim \Omega(X) = 1$.

Problem 30 Let X be a Riemann surface.

a) For $U \subset X$ open, let $\mathcal{B}(U)$ be the vector space of all bounded holomorphic functions $f : U \to \mathbb{C}$.

Show that $\mathcal{B}$, together with the natural restriction maps, is a presheaf which satisfies sheaf axiom (Sh1), but not sheaf axiom (Sh2).

b) For $U \subset X$ open, define $\mathcal{F}(U) := \mathcal{O}^*(U)/\exp \mathcal{O}(U)$.

Show that $\mathcal{F}$, together with the natural restriction maps, is a presheaf (of abelian multipicative groups) which does not satisfy sheaf axiom (Sh1).

Problem 31

Let X, S be topological spaces, $p: X \to S$ a continuous map and $\mathcal{F}$ a sheaf of abelian groups on X. For $U \subset S$ open define

$$(p_*\mathcal{F})(U) := \mathcal{F}(p^{-1}(U)).$$

a) Show that $p_*\mathcal{F}$, together with the natural restriction maps, is a sheaf of abelian groups on S. It is called the *image sheaf* of $\mathcal{F}$ with respect to p.

b) Let $\mathcal{C}_X$ (resp. $\mathcal{C}_S$) be the sheaf of continuous (complex-valued) functions on X (resp. S). Show that there is a natural homomorphism of sheaves

$$p^* : \mathcal{C}_S \to p_*\mathcal{C}_X, \qquad p^*(f) := f \circ p.$$

Problem 32

Suppose $p_1, \ldots, p_n$ are pairwise distinct points of $\mathbb{C}$ and let

$$X := \mathbb{C} \setminus \{p_1, \ldots, p_n\}.$$

Prove that $H^1(X, \mathbb{Z}) \cong \mathbb{Z}^n$.

Hint. Construct an open covering $\mathfrak{U} = (U_1, U_2)$ of X such that U_ν are connected and simply connected and $U_1 \cap U_2$ has $n + 1$ connected components.

Due: Wednesday, December 19, 2012, 15 h