LUDWIG-MAXIMILIANS-UNIVERSITÄT MÜNCHEN
MATHEMATISCHES INSTITUT

Wintersemester 2010

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Analysis III Tutorium

Lösungen

Aufgabe 11.1.

$\varphi_1 \wedge \varphi_2((a_1,a_2),(b_1,b_2)) = \begin{vmatrix} \varphi_1(a_1,a_2) & \varphi_1(b_1,b_2) \\ \varphi_2(a_1,a_2) & \varphi_2(b_1,b_2) \end{vmatrix} = \varphi_1(a_1,a_2)\varphi_2(b_1,b_2) - \varphi_2(a_1,a_2)\varphi_1(b_1,b_2) =$

$= (2a_1 - 3a_2)(b_1 + b_2) - (a_1 + a_2)(2b_1 - 3b_2) =$

$= 2a_1b_1 + 2a_1b_2 - 3a_2b_1 - 3a_2b_2 - 2a_1b_1 + 3a_1b_2 - 2a_2b_1 + 3a_2b_2 =$

$= 5a_1b_2 - 5a_2b_1$

Aufgabe 11.2.

i.

Da $\omega \wedge \omega = (-1)^{kk}\omega \wedge \omega$, dann für k ungerade $\omega \wedge \omega = -1(\omega \wedge \omega)$. Deswegen $\omega \wedge \omega = 0$.

ii.

Sei ω die 2-differentialform auf $\mathbb{R}^4$ gegeben als $\omega := \pi_1 dx_1 \wedge dx_2 + \pi_2 dx_3 \wedge dx_4$, wo $\pi_1(a,b,c,d) := a$ und $\pi_2(a,b,c,d) := b$. Dann $\omega \wedge \omega = \pi_1\pi_2 dx_1 \wedge dx_2 \wedge dx_3 \wedge dx_4 + \pi_1\pi_2 dx_3 \wedge dx_4 \wedge dx_1 \wedge dx_2 = 2\pi_1\pi_2 dx_1 \wedge dx_2 \wedge dx_3 \wedge dx_4 \neq 0$.

Aufgabe 11.3.

$\omega \wedge \eta = (xdx + ydy) \wedge (zydx + xzdy + xydz) =$

$$= [(xdx + ydy) \wedge (zydx)] + [(xdx + ydy) \wedge (xzdy)] + [(xdx + ydy) \wedge (xydz)] =$$

$$= xyz(dx \wedge dx) + zy^2(dy \wedge dx) + x^2z(dx \wedge dy) + xyz(dy \wedge dy) + x^2y(dx \wedge dz) + xy^2(dy \wedge dz) =$$

$$= -zy^2(dx \wedge dy) + x^2z(dx \wedge dy) + x^2y(dx \wedge dz) + xy^2(dy \wedge dz) =$$

$$= (x^2z - y^2z)(dx \wedge dy) + x^2y(dx \wedge dz) + xy^2(dy \wedge dz).$$

Aufgabe 11.4.

$d\eta = d(F(dx \wedge dy)) + d(G(dy \wedge dz)) + d(H(dz \wedge dx))$.

Aber $d(Fdx \wedge dy) = dF \wedge dx \wedge dy = (\frac{\partial F}{\partial x}dx + \frac{\partial F}{\partial y}dy + \frac{\partial F}{\partial z}dz) \wedge (dx \wedge dy) =$

$$= \frac{\partial F}{\partial x}dx \wedge (dx \wedge dy) + \frac{\partial F}{\partial y}dy \wedge (dx \wedge dy) + \frac{\partial F}{\partial z}dz \wedge (dx \wedge dy) =$$

$$= \frac{\partial F}{\partial x}(dx \wedge dx \wedge dy) + \frac{\partial F}{\partial y}(dy \wedge dx \wedge dy) + \frac{\partial F}{\partial z}(dz \wedge (dx \wedge dy)) =$$

$$= 0 + 0 + \frac{\partial F}{\partial z}((-1)^{1*2}(dx \wedge dy) \wedge dz) = \frac{\partial F}{\partial z}(dx \wedge dy \wedge dz).$$

Analog $d(G(dy \wedge dz)) = \frac{\partial G}{\partial x}(dx \wedge dy \wedge dz)$ und $d(H(dz \wedge dx)) = \frac{\partial H}{\partial y}(dx \wedge dy \wedge dz)$.

Somit $d\eta = (\frac{\partial F}{\partial z} + \frac{\partial G}{\partial x} + \frac{\partial H}{\partial y})(dx \wedge dy \wedge dz)$.