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Präsenzübungen

Ex. 1 Let $-\infty \leq a < b \leq +\infty$, $\phi \in C^\infty((a,b);\mathbb{R})$, $\psi \in C_c^\infty((a,b);\mathbb{C})$ s.t. $\phi'(x) \neq 0$ for all $x \in \operatorname{supp}\psi$. Then

$$I(\lambda) := \int_a^b e^{i\lambda\phi(x)}\psi(x)\,dx \quad , \quad \lambda > 0$$

satisfies $I(\lambda) = \mathcal{O}(\lambda^{-N})$ as $\lambda \to \infty$ for arbitrary $N \geq 0$ (i.e. $\forall N \geq 0\ \exists C_N > 0$ s.t. $|I(\lambda)| \leq C_N \lambda^{-N}$)

Ex. 2 Let $\phi \in C^\infty(\mathbb{R}^n;\mathbb{R})$, $\psi \in C_c^\infty(\mathbb{R}^n;\mathbb{C})$ s.t. $\nabla\phi(x) \neq 0$ for all $x \in \operatorname{supp}\psi$. Then

$$I(\lambda) := \int_{\mathbb{R}^n} e^{i\lambda\phi(x)}\psi(x)\,dx \quad , \quad \lambda > 0$$

satisfies $I(\lambda) = \mathcal{O}(\lambda^{-N})$ as $\lambda \to \infty$ for arbitrary $N \geq 0$.

Ex. 3 Let ϕ, ψ be as in Ex. 2, but assume that ϕ depends on a parameter $\lambda > 0$, i.e. $\phi = \phi(x;\lambda)$, s.t. $|\nabla\phi(x)| \geq c\lambda$ for some $c > 0$, all $\lambda \geq \lambda_0 > 0$ and all $x \in \operatorname{supp}\psi$. Then

$$\int_{\mathbb{R}^N} e^{i\phi(x;\lambda)}\psi(x)\,dx = \mathcal{O}(\lambda^{-N}) \quad \forall N \geq 0.$$

Ex. 4 Let $f \in C_c^\infty((a^2,b^2);\mathbb{C})$, $0<a<b$, $\alpha(r) := tr^2 + r\omega\cdot x - \lambda\log(r)$. Then

$$\int_0^\infty f(r^2)r^{n/2-1}e^{i\alpha(r)}\,dr = \mathcal{O}\left((1+|\lambda|+|t|)^{-N}\right) \quad \forall N \geq 0$$

holds for $t \geq 0$, $\lambda \leq 0$, $|x| \leq 2at$, $\omega \in S^{n-1}$.