22.11.17

Präsenzübungen 5

Exercise 1 Prove that $\exists\, a, b > 0$ s.t.

$$\forall \psi \in \mathcal{D}(-\Delta): \quad \| |\cdot|^{-1} \psi \| \leq a \|\psi\| + b \|-\Delta \psi\|.$$

Hint: Use Hardy's inequality: $d \geq 3$

$$\forall \psi \in H^{1,2}(\mathbb{R}^d): \int_{\mathbb{R}^d} \frac{|\psi(x)|^2}{|x|^2} \leq \left(\frac{2}{d-2}\right)^2 \int_{\mathbb{R}^d} |\nabla \psi(x)|^2 \, dx$$

Exercise 2 Prove that for $s > d/2$, $H^{s,2}(\mathbb{R}^d)$ embeds continuously into $C_0(\mathbb{R}^d)$, i.e. $H^{s,2}(\mathbb{R}^d) \subseteq C_0(\mathbb{R}^d)$ and $\exists\, C > 0$ s.t.

$$\forall \psi \in H^{s,2}(\mathbb{R}^d): \quad \sup_{x \in \mathbb{R}^d} |\psi(x)| \leq C \|\psi\|_{H^{s,2}(\mathbb{R}^d)}.$$

Hint: Consider $\|\hat{\psi}\|_{L^1}$ and use Cauchy-Schwarz. Apply the Riemann-Lebesgue lemma.