

## Compact operators

Exercise 1 Let  $T \in BL(\mathcal{H})$  be compact. Prove:

$$\psi_n \xrightarrow{n \rightarrow \infty} \psi \text{ weakly} \Rightarrow T\psi_n \xrightarrow{n \rightarrow \infty} T\psi \text{ strongly}$$

Exercise 2 Let  $(T_n)_{n \in \mathbb{N}} \subset BL(\mathcal{H})$  compact and  $T \in BL(\mathcal{H})$ . Prove:

$$\|T_n - T\| \xrightarrow{n \rightarrow \infty} 0 \Rightarrow T \text{ compact.}$$

Exercise 3 Let  $T \in BL(\mathcal{H})$ . Prove

$$T \underset{(i)}{\text{compact}} \Leftrightarrow T^* T \underset{(ii)}{\text{compact}} \Leftrightarrow T^* \underset{(iii)}{\text{compact}}$$

Exercise 4 Let  $T \in \mathcal{I}^p(\mathcal{H})$ . Then  $T^* T \in \mathcal{I}^{p/2}(\mathcal{H})$ .