17.1.18

Präsenzübungen

Ex. 1 Let $n \geq 1$, $\phi : \mathbb{R}^n \to \mathbb{R}$ smooth and $\psi : \mathbb{R}^n \to \mathbb{C}$ smooth and compactly supported. Define

$$I(\lambda) := \int_{\mathbb{R}^n} e^{i\lambda\phi(x)} \psi(x)\, dx \quad , \quad \lambda > 1 .$$

Prove: If $\nabla\phi(x) \neq 0$ on $\operatorname{supp}\psi$, then $I(\lambda) = O(\lambda^{-N})$ for all $N \geq 0$.

Ex 2 Let $n = 1$, $\phi, \psi, I(\lambda)$ as above. Assume:

$$\left.\begin{array}{l} x \in \operatorname{supp}\psi \\ \phi'(x) = 0 \end{array}\right\} \Longrightarrow \phi''(x) \neq 0 .$$

Prove: $I(\lambda) = O(\lambda^{-\frac{1}{2}})$

Ex. 3 Prove the higher-dimensional version of Ex. 2, i.e.

THM (stationary phase)

Let $n \geq 1$, $\phi : \mathbb{R}^n \to \mathbb{R}$ smooth and $\psi : \mathbb{R}^n \to \mathbb{C}$ smooth and compactly supported. Assume:

$$\left.\begin{array}{l} x \in \operatorname{supp}\psi \\ \nabla\phi(x) = 0 \end{array}\right\} \Longrightarrow \det D^2\phi(x) \neq 0 .$$

Then $I(\lambda) = O(\lambda^{-n/2})$.