Mathematisches Institut
LMU München
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Mathematical Quantum Mechanics

Problem Sheet 13

Hand-in deadline: 03/03/2017 before noon in the designated MQM box (1st floor, next to the library).

Ex. 1: Determine whether the following potentials in $\mathbb{R}^3$ are dilation-analytic,

$$\mathbf{1}\{|x| \leq 1\}, \quad \frac{1}{1+|x|^2}, \quad \frac{1}{1+x_1^2+x_2^4+x_3^6}, \quad e^{-|x|^2}, \quad \frac{\sin(|x|)}{|x|}.$$

More precisely, determine for which $\alpha > 0$ these potentials belong to $\mathcal{F}_\alpha$.

Ex. 2: Consider $H_0 = -d^2/dx^2$ on $L^2(\mathbb{R})$ and a rank one perturbation $H_\epsilon = H_0 + \epsilon|\psi\rangle\langle\psi|$. Here $\psi \in C_c^\infty(\mathbb{R})$ is such that $\int \psi \neq 0$ and $\epsilon \in \mathbb{R} \setminus \{0\}$.

1. Derive the resolvent formula

$$(H_\epsilon - k^2)^{-1} = (H_0 - k^2)^{-1} - \epsilon \frac{|(H_0-k^2)^{-1}\psi\rangle\langle(H_0-k^2)^{-1}\psi|}{1+\epsilon\langle\psi,(H_0-k^2)^{-1}\psi\rangle}$$

   for $k \in \mathbb{C}$.

2. Prove that for any $f, g \in C_c^\infty(\mathbb{R})$, the functions $k \mapsto \langle f, (H_0-k^2)^{-1}g\rangle$ and $k \mapsto \langle f, (H_\epsilon - k^2)^{-1}g\rangle$ are meromorphic, and find the eigenvalues and resonances of H_ϵ in the weak-coupling limit $\epsilon \to 0$.