Ludwig-Maximilians-Universität München
Prof. Dr. Thomas Vogel

Seminar Sommersemester 2015
h-Prinzipien

In diesem Seminar behandeln die Lösbarkeit von Fragen der folgenden Art: Gegeben sei eine Mannigfaltigkeit M der Dimension n. Gibt es eine Immersion $f : M \longrightarrow \mathbb{R}^n$?

Die Antwort auf diese Frage ist im Allgemeinen nein, zum Beispiel muss M orientierbar sein. Wenn M orientierbar und geschlossen ist, gibt es auch keine solche Immersion. Wenn M dagegen nicht kompakt ist, dann kann man dagegen vergleichsweise einfach feststellen, ob eine solche Immersion existiert. Insbesondere gibt es effektive Methoden, Immersionen zu konstruieren. Wir werden hauptsächlich solche Konstruktionsmethoden in einem allgemeinem Rahmen behandeln.

Eine berühmte Anwendung ist folgender Satz von Smale: Man kann die 2-Sphäre in $\mathbb{R}^3$ so durch Immersionen deformieren, dass die Innen- und die Außenseite vertauscht werden.

Eine solche Familie von Immersionen kann schön visualisiert werden, etwa im Film *Inside out*.

Vom 22.-26. Juni findet an der LMU ein Workshop mit einem eng verwandten Thema statt statt. Mehr Informationen dazu finden Sie auf

http://www.math.lmu.de/~bowden/Wrinkles/Wrinkles.php.

Zielgruppe: Seminar für Studenten in Master Mathematik und mutige Bachelorstudenten. TMP Studenten sind willkommen.

Literatur: Y. Eliashberg, N. Mishachev, *Introduction to the h-principle*, Grad. Studies in Math. Vol 48 (AMS 2002)
Y. Eliashberg, N. Mishachev, *Wrinkling of smooth mappings and its applications I*, Invent. Math. 130 (1997), no. 2, 345–369
M. Golubitsky, V. Guillemin, *Stable mappings and their singularities*, Graduate Texts in Mathematics, Vol. 14. Springer-Verlag (1973)

Vorkenntnisse: Unabdingbar sind Mannigfatigkeiten, Tangentialbündel, Vektorfelder und Flüsse. Sehr hilfreich wären noch etwas algebraische Topologie, insbesondere Homotopien und Differentialformen.

Termin/Ort: Dienstag, 12-14 Uhr im Hörsaal B252

Sprache: Wir noch festgelegt, die Literatur ist in englisch.

Einleitung und Vortragsvergabe am 14.4.2015, 12-14 Uhr in B252

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Seminar Sommersemester 2015
h-principles

In this seminar we study the solvability of problems of the following type: Let M be a manifold of dimension n. Does there exist an immersion $f : M \longrightarrow \mathbb{R}^n$?

The answer is no in general. For example, M has to be orientable. When M is orientable but closed, then there is no immersion $f : M \longrightarrow \mathbb{R}^n$, either (by the theorem of Stokes). In contrast, when M is open, then it is comparatively easy to determine whether or not such an immersion exists. In particular, there are effective methods for the construction of immersions. The main topic of this seminar are similar (but more general) constructions.

A famous application is the following theorem of Smale: It is possible to deform the 2-sphere in $\mathbb{R}^3$ through immersions, so that the inner and the outer side of the sphere are interchanged.

There are beautiful visualizations of such a family of immersions, eg. the movie *Inside out*.

From June 22 until June 26 the LMU will host a workshop on a closely related topic. More information can be found on

http://www.math.lmu.de/~bowden/Wrinkles/Wrinkles.php.

Audience: Seminar for master students in mathematics and courageous bachelor students. TMP students are welcome.

Literature: Y. Eliashberg, N. Mishachev, *Introduction to the h-principle*, Grad. Studies in Math. Vol 48 (AMS 2002)
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Prerequisites: Manifolds, tangent bundles, vector fields and flows are indispensable. Some basic algebraic topology, like homotopies and differential forms, are very helpful.

Time/Place: Tuesday, 12-14 in Room B252

Language: To be determined in the first meeting.

Organisational meeting on April 14, 2015, 12-14 in B252

h-principles
Preliminary program

1. **Jets, holonomic sections of jet bundles**
   Introduce jet spaces and the notion of holonomic section, give the definition of vector bundles, sections. Relationship between 1-jets of real valued functions and 1-forms. Whitney topology on spaces of maps (the C^r-topologies are good enough for us (no need for C^∞).
   Date: April 21
   Literature: Chapter 1 of [ElM], p.7–14, or Chapter II.2 of [GoG], p.37–42, Chapter II.3 of [GoG], p. 42–50.
   Speaker: T.V.
2. **Thom transversality theorem (strong version), Sard's Lemma**
   Prove at least the simplest versions of Sard's lemma and then Thom's strong transversality theorem.
   Date: April 28
   Literature: Chapter 2 of [ElM], p.15–20 (or Chapter II.4 of [GoG], p. 50–59), Chapter II.1 of [GoG], p.30–37.
   Speaker: T.V.
3. **Holonomic Approximation I**
   Formulate the non-parametric holonomic approximation theorem, fiberwise holonomic sections, proof of the inductive lemma.
   Date: May 5
   Literature: first half of Chapter 3 of [ElM], p. 21–31.
   Speaker: B.H.
4. **Holonomic Approximation II**
   Similar to the previous talk, but with parameters. This requires some repetitions from the previous talk. Smale's theorem on sphere eversions.
   Date: May 12
   Literature: second half of Chapter 3 and beginning of Chapter 4 in [ElM] , p. 31–39.
   Speaker: T.V.
5. **Applications, Differential relations**
   Your choice of application from Chapter 4, discuss differential relations (open, closed) and (formal versus genuine) solutions of differential relations.
   Date: May 19
   Literature: remainder of Chapter 4, Chapter 5 of [ElM] p. 40–58.
   Speaker:
6. **Open** $\mathrm{Diff}(M)$**-invariant relations, Applications to closed manifolds**
   Date: May 26 (Pfingstdienstag, wir **brauchen** einen Ersatztermin)
   Literature:
   Speaker: N.P.
7. **Wrinkled maps I**
   Notion of wrinkles, wrinkled map, fibered wrinkles, statement of Theorem 1.5A, 1.6A, cite Philips' submersion theorem to prove Theorem 1.6A (all in [ElM2])
   Date: June 2
   Literature: [ElM2], p. 346–357.
   Speaker: R.C. and G.P.

8. **Wrinkled maps II**
   Explain as much as possible from the proof of Theorem 1.5A in [ElM2]
   Date: June 9
   Literature: [ElM2], p.357–368.
   Speaker: R.C. and G.P
9. **no seminar**
   Date: June 16
10. **Workshop**
   Date: June 22 - June 26
11. **Convex integration**
   One-dimensional convex integration.
   Date: June 30
   Literature: Chapter 17 of [ElM], p. 153–165, or Lecture 1 of [Bo]
   Speaker: P.F.
12. **The theorem of Nash-Kuiper**
   Existence of isometric C^1-immersions for compact manifolds.
   Date: July 7
   Literature: Chapter 21 of [ElM], p. 189–197 or Lecture 3 from Borelli's notes [Bo].
   Speaker:
13. **Additional topic: Flat tori in $\mathbb{R}^3$**
   Apply the proof of Nash-Kuiper's theorem to describe embeddings of flat tori in $\mathbb{R}^3$.
   Date: July 14
   Literature: Lecture 4 from V. Borelli's notes [Bo]
   Speaker: T.K.

Literatur

[Bo] V. Borelli, *Lecture notes from a seminar in Les Diablerets, 2012*, available at *http://www.homepages.ucl.ac.uk/ ucahcwe/h-principle.html*.

[ElM] Y. Eliashberg, N. Mishachev, *Introduction to the h-principle*, Grad. Studies in Math. Vol 48 (AMS 2002).

[ElM2] Y. Eliashberg, N. Mishachev, *Wrinkling of smooth mappings and its applications I*, Invent. Math. 130 (1997), no. 2, 345–369

[GoG] M. Golubitsky, V. Guillemin, *Stable mappings and their singularities*, Graduate Texts in Mathematics, Vol. 14. Springer-Verlag (1973)