LMU LUDWIG-MAXIMILIANS-UNIVERSITÄT MÜNCHEN
MATHEMATISCHES INSTITUT

WiSe 2018/19

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Symplectic geometry

Exercise sheet 4

Exercise 1. Let $(M, \omega = d\lambda)$ be an exact symplectic manifold and $f : L \longrightarrow M$ a Lagrangian immersion. The *symplectic area class* is defined as $[f^*\lambda] \in H^1_{dR}(L)$.

Show that this class depends only on ω but not on the choice of primitive λ when $H^1_{dR}(M) = 0$. Let Σ be a surface and $s : \Sigma \longrightarrow M$ be a smooth map such that $s(\partial\Sigma) \subset L$. Compute $\int_\Sigma s^*\omega$ in terms of the symplectic area class.

Exercise 2. Let $L = \{x_i^2 + y_i^2 = 1 \mid \text{ for all } i = 1, \ldots, n\} \subset \mathbb{C}^n$. Show that L is a embedded Lagrangian torus and compute $\|\mu\|$.

Exercise 3. Let (M, ω) be a connected symplectic manifold and $x, y \in M$. Show that there is Hamiltonian symplectomorphism ψ of (M, ω) so that $\psi(x) = y$.

Hint: Solve the exercise first for balls in $(\mathbb{R}^{2n}, \omega_{st})$ and use Darboux theorem.

Exercise 4. Let M be a smooth manifold.

a) For α any form and vector fields X, Y on M show that

$$L_X i_Y \alpha - i_Y L_X \alpha = i_{[X,Y]}\alpha.$$

Hint: Assume that α is a 1-form. Then use that every k-form can be written as sum of products of 1-forms.

b) Assume that (ω, M) is symplectic and that X, Y are Hamiltonian vector fields. Prove that $[X, Y]$ is also Hamiltonian. This implies that the Hamiltonian vector fields form a Lie-algebra.

Hand in on Wednesday November, 14 during the exercise class.