LUDWIG-MAXIMILIANS-UNIVERSITÄT MÜNCHEN

MATHEMATISCHES INSTITUT

WiSe 2018/19

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Symplectic geometry

Exercise sheet 13

Exercise 1. In the setting of Thursdays lecture, prove that if $x(t)$ is a 1-periodic solution of $\dot{x}(t) = J\nabla\overline{H}(x(t))$ with $|x(0)| > r$, then $\overline{\Phi}(x) \leq 0$. Show in addition, that if $x(t)$ is constant, then $\Phi(x) \leq 0$.

Exercise 2. In the same setting, show that $\overline{\Phi}(x) \leq b$ for all $x \in \Sigma_\tau$ and that $\overline{\Phi} \leq 0$ on $\partial\Sigma_\tau$ for τ large.

Exercise 3. Let $J = \begin{pmatrix} 0 & -\mathrm{id} \\ \mathrm{id} & 0 \end{pmatrix}$ and consider the Liouville form $\lambda = \frac{1}{2}\sum_i (x_i dy_i - y_i dx_i)$ on $\mathbb{R}^{2n}$. Show that

$$\int_0^1 \frac{1}{2}\langle -J\dot{z}(t), z(t)\rangle dt = \int_{S^1} z^*\lambda$$

for a smooth loop $z = (x, y) : S^1 = \mathbb{R}/\mathbb{Z} \longrightarrow \mathbb{R}^{2n}$.

Exercise 4. Assume that X is a Liouville vector field on $\mathbb{R}^{2n}$ which is transverse to the closed hypersurface S. Show that $\alpha = i_X\omega$ is a contact form on S, i.e. $\alpha \wedge (d\alpha)^{n-1}$ vanishes nowhere.

Hand in on Wednesday January 23 during the exercise class.