WiSe 2018/19

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Symplectic geometry

Exercise sheet 10

Exercise 1. Let $U_1, \dots U_n$ be an open cover of S^n by balls. Show that at least one of the sets contains a pair of antipodal points. Argue be contradiction and use the covering $S^n \longrightarrow \mathbb{RP}^n$.

You may use the fact that the cup-length of $\mathbb{RP}^n$ is $n+1$.

Exercise 2. Let L be the zero section of T^*S^1 with the standard symplectic structure and ϕ a Hamiltonian isotopy of T^*S^1. Prove that

$$|L \cap \phi(L)| \geq 2^1$$

Remark: This statement remains true when S^1 is replaced by T^n and 2^1 is replaced by 2^n.

Exercise 3. Let (M, ω) be a symplectic manifold, $L \subset M$ a Lagrangian submanifold and $\phi_t \in \mathrm{Ham}(M, \omega), t \in S^1$, a loop of Hamiltonian diffeomorphisms generated by the smooth family of functions $H_t, t \in S^1$.

One T^*S^1 we consider the standard symplectic forms and the coordinates $(r, t) \in \mathbb{R} \times S^1 \simeq T^*S^1$. Show that

$$\begin{aligned} F : L \times S^1 &\longrightarrow M \times T^*S^1 \\ (l, t) &\longmapsto (\phi_t(x), -H_t(\phi_t(x)), t) \end{aligned}$$

is a Lagrangian in $(M \times T^*S^1, \sigma = \omega + dr \wedge dt)$.

Hand in on Wednesday January 9 during the exercise class.
Merry Christmas + Happy new year