Smoothed Particle Hydrodynamics

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Smoothed Particle Hydrodynamics

- numerical method to simulate fluids (liquids, gases, plasmas)
- idea: represent fluid by moving particles
- first used in astrophysics
- increasingly used in CGI for block-buster movies
- upcoming technology for next-generation computer games

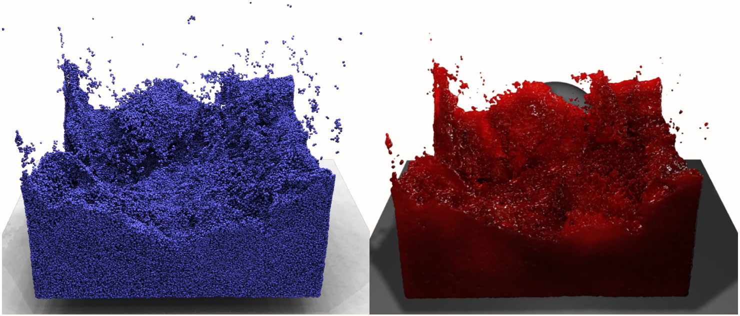

Figure: 1 million particles, rendered in Maya, by Frank Zimmer

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4. Note that $A(r) = (A * \delta)(r) = \int A(r')\delta(r - r')\mathrm{d}r'$
5. Approximate $\delta(r - r')$ by $W_h(r - r')$, where
   - $\int W_h(r')\mathrm{d}r' = 1$
   - $W_h \overset{*}{\rightharpoonup} \delta$ for $h \to 0$
   - W_h radially symmetric
   - $W_h \in C_0^\infty$
   - supp $W_h \subset B_h(0)$

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Importance of smoothing length

- The summation in

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- Defining the neighbourhood of particle i as

$$\mathcal{N}_i := \{1 \le j \le N | r_j \in B_h(r_i)\}$$

one can write

$$[A]_i = \sum_{j \in \mathcal{N}_i} m_j \frac{A_j}{\rho_j} W_h(r_i - r_j).$$

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$$(\nabla [A])\,(r) = \sum_{j=1}^{N} m_j \frac{A_j}{\rho_j} (\nabla W_h)(r - r_j)$$

- Laplacian:

$$(\Delta [A])\,(r) = \sum_{j=1}^{N} m_j \frac{A_j}{\rho_j} (\Delta W_h)(r - r_j)$$

Spatial derivatives

- SPH Field approximation

$$A(r_i) \approx [A]_i := \sum_{j \in \mathcal{N}_i} m_j \frac{A_j}{\rho_j} W_h(r_i - r_j)$$

- SPH Gradient approximation:

$$(\nabla A)(r_i) \approx \nabla [A]_i = \sum_{j \in \mathcal{N}_i} m_j \frac{A_j}{\rho_j} (\nabla W_h)(r_i - r_j)$$

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Practical considerations

- Smoothing length h proportional to average particle diameter:

$$h \sim \frac{1}{\langle \rho \rangle^{\frac{1}{d}}}, \text{ where } \langle \rho \rangle := \frac{1}{n} \sum_{i=1}^{N} \rho_i$$

- Different kernels suitable for different charge densities.
- Kernels not C^∞ due to performance considerations (Splines!).
- Golden rules of SPH (Monaghan):
  - To find physical interpretation it's always best to assume kernel is Gaussian.
  - Rewrite formulas with mass density inside operators, by making use of

$$\nabla A = \frac{\nabla(A\psi)}{\psi} - \frac{A(\nabla\psi)}{\psi}$$

for positive smooth ψ.

Improved gradients

- Consider $A(r) = \text{const}$. Then $(\nabla A)(r) = 0$, but

$$(\nabla A)(r_i) \approx \nabla[A]_i = \sum_{j \in \mathcal{N}_i} m_j \frac{A_j}{\rho_j} (\nabla W_h)(r_i - r_j) \neq 0.$$

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- Using $\nabla A = \frac{\nabla(A\psi) - A(\nabla\psi)}{\psi}$ leads to

$$\begin{aligned}(\nabla A)(r_i) &\approx \frac{\nabla[A\psi]_i - A_i \nabla[\psi]_i}{\psi_i} \\ &= \frac{1}{\psi_i} \sum_{j \in \mathcal{N}_i} m_j \frac{(A_j - A_i)\psi_j}{\rho_j} (\nabla W_h)(r_i - r_j) = 0.\end{aligned}$$

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- For example $\psi = \rho$:

$$(\nabla A)(r_i) \approx \frac{1}{\rho_i} \sum_{j \in \mathcal{N}_i} m_j (A_j - A_i)(\nabla W_h)(r_i - r_j)$$

Equations of motion

- Navier-Stokes equation:

$$\rho\left(\partial_t v + v \cdot \nabla v\right) = \rho g - \nabla p + \mu \Delta v, \text{ where}$$

  - v velocity
  - g gravity
  - p pressure
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  will be trivially satisfied: each particle has constant mass and particles are neither created nor destroyed.

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$$\frac{\mathrm{d}}{\mathrm{d}t} v(r,t) = (\partial_t v)(r,t) + [\dot{r}(t)] \cdot (\nabla v)(r,t)$$

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- Velocity v_i of particle moving with the fluid, i.e. $\dot{r}_i = v_i$:

$$\frac{\mathrm{d}}{\mathrm{d}t} v_i = \partial_t v_i + v_i \cdot \nabla v_i,$$

  i.e. Navier-Stokes can be seen as Newton's second law in disguise.

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Pressure term

- Naive SPH-approximation of ∇p would yield

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Viscosity term

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- Using $\nabla v = \frac{\nabla(v\psi)}{\psi} - \frac{v(\nabla\psi)}{\psi}$ with $\psi = 1$ yields

$$\begin{aligned}\frac{\mu}{\rho_i}(\Delta v)_i &\approx \frac{\mu}{\rho_i}\left(\Delta[v]_i - v\Delta[1]_i\right)\\ &= \frac{\mu}{\rho_i}\sum_{j\in\mathcal{N}_i}\frac{v_j - v_i}{\rho_j}(\Delta W_h)(r_i - r_j)\end{aligned}$$

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- Modelled as

$$p_i = k(\rho_i - \rho_{\text{eq}}), \text{ where}$$

  - k constant depending on temperature
  - ρ_{eq} equilibrium density (set to zero for ideal gas)

Algorithm

For each timestep:

1. For each particle: compute density and pressure

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3. For each particle: integrate position and velocity with symplectic Euler scheme.

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- Different types of boundaries:
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  - Mixtures of Slip/Noslip
  - Pressure boundaries
  - Flux boundaries
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  - Reflective boundaries
- Different methods to model boundaries:
  - Boundary particles
  - Ghost particles
  - Virtual forces
  - Analytical methods

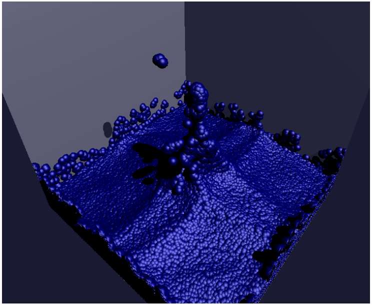
Advanced topics

Active areas of research in SPH include:

- Boundary modelling
- Adaptivity
- Surface tension
- Solid adhesion

Demo

- 64k particles, interactive frame-rates
- Graphics running against DirectX 11 (Windows only)
- Simulation running against OpenCL (Windows, Linux, Android, Supercomputers...)
- Surface tension and solid adhesion modelled according to Akinci, Akinci and Teschner (2013), Freiburg



Thanks for your attention!

Please do not hesitate to ask questions!