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Wintersemester 17/18
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Logik

Blatt 14 (Probeklausur)

Aufgabe 1. Let $\mathcal{M} = (D, F, \mathbf{i}, \mathbf{j})$ be a fan model, t a term, and η, ξ assignments in D. Show that if $\eta(x) = \xi(x)$, for all $x \in \mathrm{FV}(t)$, then

$$\eta_{\mathcal{M}}(t) = \xi_{\mathcal{M}}(t).$$

(6 points)

Aufgabe 2. Find an appropriate condition on formula A or on formula B such that the formula

$$\exists_x(A \to B) \to (A \to \exists_x B)$$

is derivable in minimal logic, and write such a derivation. (6 points)

Aufgabe 3. (i) A^g is the Gödel-Gentzen translation of formula A.
Prove or disprove the following formula:

$$\forall_{A,B \in \texttt{Form}}(A^g = B^g \Rightarrow A = B).$$

(2 points)

(ii) Let X be an inhabited set. All relations on X considered next are of arity 2.
A relation R on X is called symmetric, if

$$\forall_{x,y \in X}\big(R(x,y) \Rightarrow R(y,x)\big).$$

(a) Give an example of a symmetric relation on X. (1 point)

(b) If R is a relation on X, define the symmetric closure R^s of R and write the induction principle that corresponds to this definition. (1+2 points)

Aufgabe 4. (i) The relation $B \lhd^{\oplus} A$, "B is a strictly positive Gentzen subformula of A" is defined in Definition 3.5.2.

Let $B_1, \ldots, B_k \in \texttt{Form}$, for some $k > 0$, and let $A \equiv B_1 \to B_2 \to \ldots \to B_k$. If

$$\begin{aligned} A_0 &\equiv A, \\ A_1 &\equiv B_2 \to \ldots \to B_k, \\ &\ldots \\ A_{k-2} &\equiv B_{k-1} \to B_k, \\ A_{k-1} &\equiv B_k, \end{aligned}$$

show that $A_j \lhd^{\oplus} A$, for every $j \in \{0, \ldots, k-1\}$. (1 point)

(ii) Let $A_1, \ldots, A_n, B, C \in \texttt{Form}$ such that

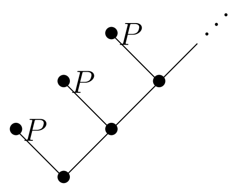
$$\forall_{i \in \{1,\ldots,n\}} \big(\neg(B \vee C) \lhd^{\oplus} A_i\big).$$

Show that there is no derivation

$$A_1, \ldots, A_n \vdash B \vee C$$

within the $(\to)$-framework of minimal logic. (5 points)

Aufgabe 5. Let $\mathcal{L} = (\{\bot, P, Q\}, \emptyset)$ such that $P, Q \in \texttt{Rel}^{(0)}$. Let F be the following subfan of the Cantor tree, where next to every node we write all relations of $\mathcal{L}$ forced at that node.



(i) If D is any inhabited set, show that the fan model $M_i = (D, F, \mathbf{i}, \mathbf{j})$ determined by the above diagram is an intuitionistic fan model. (2 points)

(ii) Find η, u such that $(\mathcal{M}_i, \eta, u)$ is an intuitionistic countermodel to the derivation

$$\vdash_i ((P \to Q) \to P) \to P.$$

(4 points)

Aufgabe 6. Let $\mathcal{L}$ be a countable first-order language, and let Γ be a set of formulas in $\mathcal{L}$. Show that if Γ is satisfiable, then Γ is satisfiable in a (classical) model $\mathcal{M}_c$ with a countably infinite carrier set $|\mathcal{M}_c|$. (6 points)

Hint: Work in a classical metatheory, and take into account Remark 4.4.8.

Besprechung. Montag, 05. Februar 2018, in der Vorlesung.