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Sommersemester 18
14.06.2018

Gewöhnliche Differentialgleichungen

Blatt 9

Aufgabe 1. Let $A \in M_2(\mathbb{R})$ such that its eigenvalues are $\lambda = ib$ and $\mu = -ib$, for some $b > 0$. Show that all solutions of the system $\dot{x}(t) = Ax(t)$ are periodic with the same period.

Aufgabe 2. Let $A \in M_n(\mathbb{R})$ such that $\mathcal{T}_A \in L(\mathbb{R}^n)$ leaves the subspace $E \subseteq \mathbb{R}^n$ invariant. Show that if $x(t)$ is a solution to the system $\dot{x}(t) = Ax(t)$ such that $x(t_0) \in E$, for some $t_0 \in \mathbb{R}$, then $x(t) \in E$, for every $t \in \mathbb{R}$.

Aufgabe 3. Find the general solution to the following system of odes

$$\begin{aligned}\dot{x}_1(t) &= -x_2(t),\\ \dot{x}_2(t) &= x_1(t) + t.\end{aligned}$$

Aufgabe 4. (i) Solve the following initial-value problem:

$$\ddot{s}(t) + 2\dot{s}(t) + s(t) = 0; \qquad s(0) = 1,\ \dot{s}(0) = 2.$$

(ii) Find the general solution to the ode of the harmonic oscillator

$$\ddot{s}(t) + b^2 s(t) = 0.$$

(iii) Find the general solution of the following ode:

$$\ddot{s}(t) + s(t) = t - 1.$$

(iv) Let $a_1, a_2 \in \mathbb{R}$ such that the roots of the equation $\lambda^2 + a_1\lambda + a_2 = 0$ have negative real parts. Show that if $s(t)$ is a solution to $\ddot{s}(t) + a_1\dot{s}(t) + a_2 s(t) = 0$, then it satisfies

$$\lim_{t\to\infty} s(t) = 0.$$

Abgabe. Donnerstag, 21. Juni 2018 in der Vorlesung.

Besprechung. Montag, 25. Juni 2018, in der Übung.