LMU LUDWIG-MAXIMILIANS-UNIVERSITÄT MÜNCHEN

MATHEMATISCHES INSTITUT

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Gewöhnliche Differentialgleichungen

Blatt 8

Aufgabe 1. Solve on $\mathbb{R}^3$ the system of odes $\dot{x}(t) = Ax(t)$, where

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & -3 \\ 1 & 3 & 2 \end{bmatrix}.$$

Aufgabe 2. Let $a, b \in \mathbb{R}$. If

$$A = \begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix},$$

show that the matrix of $\mathtt{e}^{T_A}$ is

$$\begin{bmatrix} e^a & 0 \\ 0 & e^b \end{bmatrix}.$$

Aufgabe 3. If $T \in L(\mathbb{R}^n)$, $\lambda \in \mathbb{R}$, $x \in \mathbb{R}^n$, and E is a subspace of $\mathbb{R}^n$, show the following:
(i) If λ is an eigenvalue of T and x is an eigenvector of T that belongs to λ, then x is an eigenvector of $\mathtt{e}^T$ that belongs to e^λ.
(ii) If E is T-invariant, then E is $\mathtt{e}^T$-invariant.

Aufgabe 4. If $L(\mathbb{R}^n)^{-1}$ is the set of all invertible operators in $L(\mathbb{R}^n)$, the following hold:
(i) The map $\mathtt{exp} : L(\mathbb{R}^n) \to L(\mathbb{R}^n)$, defined by $T \mapsto \mathtt{e}^T$, is a map from $L(\mathbb{R}^n)$ to $\mathrm{L}(\mathrm{R}^n)^{-1}$.
(ii) The map $\mathtt{exp}$ is continuous.
(iii) If $T \in L(\mathbb{R}^n)$ such that $||T|| < 1$, then
(a) the series $\sum_{k=0}^\infty T^k$ converges,
(b) $I_n - T \in \mathrm{L}(\mathrm{R}^n)^{-1}$, and

$$\sum_{k=0}^\infty T^k = \frac{1}{I_n - T}.$$

(iv) The set $\mathrm{L}(\mathrm{R}^n)^{-1}$ is an open subset of $L(\mathbb{R}^n)$.

Abgabe. Donnerstag, 14. Juni 2018 in der Vorlesung.

Besprechung. Montag, 18. Juni 2018, in der Übung.