LMU LUDWIG-MAXIMILIANS-UNIVERSITÄT MÜNCHEN
MATHEMATISCHES INSTITUT

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Gewöhnliche Differentialgleichungen

Blatt 7

Aufgabe 1. (i) Show that $(\mathbb{R}^n)_{\mathbb{C}} = \mathbb{C}^n$.
(ii) Let F be the complex subspace of $\mathbb{C}^2$ defined as the following linear span:

$$F := < \{(1, i)\} >_{\mathbb{C}} .$$

Find $F_{\mathbb{R}}$ and $(F_{\mathbb{R}})_{\mathbb{C}}$.
(iii) Find a complex subspace of $\mathbb{C}^{2018}$, which is not *-invariant.

Aufgabe 2. If E is a real subspace of $\mathbb{R}^n$, $\mathcal{B} = \{e_1, \ldots, e_m\}$ is a basis for E, $T \in L(E)$, and $\lambda \in \mathbb{C}$, show the following:
(i) $\mathcal{B}$ is a basis for $E_{\mathbb{C}}$.
(ii) The definition of the complexification $T_{\mathbb{C}}$ of T is independent from the choice of representation of $w \in E_{\mathbb{C}}$.
(iii) If $B \in M_m(\mathbb{R})$ is the matrix of T with respect to $\mathcal{B}$, then B is the matrix of $T_{\mathbb{C}}$ with respect to $\mathcal{B}$.
(iv) $p_T(\lambda) = p_{T_{\mathbb{C}}}(\lambda)$.
(v) λ is an eigenvalue of T iff λ is an eigenvalue of $T_{\mathbb{C}}$.

Aufgabe 3. Show the following:
(i) A complex subspace F of $\mathbb{C}^n$ is decomplexifiable iff F is *-invariant.
(ii) If E is a real subspace of $\mathbb{R}^n$ and $S \in L(E_{\mathbb{C}})$, then S is decomplexifiable iff S is *-preserving.

Aufgabe 4. Let the matrix

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & -3 \\ 1 & 3 & 2 \end{bmatrix}.$$

Using the operator $\mathcal{T}_A \in L(\mathbb{R}^3)$, find complex subspaces F_r, F_c of $\mathbb{C}^3$ such that $\mathbb{C}^3 = F_r \oplus F_c$.

Abgabe. Donnerstag, 07. Juni 2018 in der Vorlesung.

Besprechung. Montag, 11. Juni 2018, in der Übung.