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Sommersemester 18
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Gewöhnliche Differentialgleichungen

Blatt 5

Aufgabe 1. Show that if $\lambda_1, \dots, \lambda_n \in \mathbb{R}$ are distinct, the solution of the system in Remark 1.4.16 is a special case of the solution of the system in Theorem 1.4.17.

Aufgabe 2. Find the general solution to the system

$$\begin{aligned} \dot{x}_1(t) &= x_1(t), \\ \dot{x}_2(t) &= x_1(t) + 2x_2(t), \\ \dot{x}_3(t) &= x_1(t) - x_3(t). \end{aligned}$$

Aufgabe 3. Let $A \in M_n(\mathbb{R})$ with n distinct, real eigenvalues $\lambda_1, \dots, \lambda_n$. Find a condition P on $\lambda_1, \dots, \lambda_n$ such that the following are equivalent:
(i) $P(\lambda_1, \dots, \lambda_n)$.
(ii) For every solution $x(t)$ to the system $\dot{x}(t) = Ax(t)$ it holds

$$\lim_{t \to \infty} x(t) = 0.$$

(You also need to show that (i) and (ii) are equivalent.)

Aufgabe 4. Let $A \in M_n(\mathbb{R})$ with n distinct, real eigenvalues. We define the function

$$\phi_A : \mathbb{R} \times \mathbb{R}^n \to \mathbb{R}^n$$

$$\phi_A(t, u) = x(t),$$

where $x(t)$ is the unique solution of the system

$$\dot{x}(t) = Ax(t) \ ; \quad x(0) = u.$$

Let $t \in \mathbb{R}$ be fixed. We define

$$\phi_{A,t} : \mathbb{R}^n \to \mathbb{R}^n$$

$$\phi_{A,t}(u) = \phi_A(t, u).$$

Without using Theorem 1.4.18 show that

$$\lim_{u \to u_0} \phi_{A,t}(u) = \phi_{A,t}(u_0).$$

Abgabe. Donnerstag, 24. Mai 2018 in der Vorlesung.

Besprechung. Montag, 04. Juni 2018, in der Übung.