LUDWIG-
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UNIVERSITÄT
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# Gewöhnliche Differentialgleichungen

## Blatt 4

**Aufgabe 1.** Let $P$ be a planet moving in the Newtonian gravitational field (of the sun placed at the origin) on $U_0^{(2)}$. If the angular momentum $h$ along a solution curve $s(t)$ of $\ddot{x}(t) = m^{-1}F(x(t))$ is non-zero, then the angular momentum $h$, the total energy $E$ and the mass of $P$ satisfy

$$E \geq -\frac{m}{2h^2}.$$

**Aufgabe 2.** Let $a \in \mathbb{R}$ and $x : J \to \mathbb{R}$ be a differentiable function in the ode

$$\dot{x}(t) = ax(t). \tag{1}$$

(i) The set of solutions of (1) is the set

$$\big\{s : J \to \mathbb{R} \mid \exists_{C\in\mathbb{R}}\forall_{t\in J}\big(s(t) = Ce^{at}\big)\big\}.$$

(ii) There is a unique solution of (1) satisfying the initial condition $s(t_0) = s_0$, where $t_0 \in J$.
(iii) If $s_1, s_2$ are solutions of (1) and $\lambda_1, \lambda_2 \in \mathbb{R}$, then $\lambda_1 s_1 + \lambda_2 s_2$ is a solution of (1).
(iv) What is the dimension of the vector space of the solutions of (1)?

**Aufgabe 3.** (i) Define the vector field associated to the ode

$$\dot{x}(t) = -\frac{1}{2}x(t) \tag{2}$$

and describe it geometrically.
(ii) Define the dynamical system $\phi$ on the state space $\mathbb{R}$ that is generated by equation (2) and the history function $\phi_s$ of some $s \in \mathbb{R}$.
(iii) What can you say about the future of $s$ when time goes to $+\infty$?

**Aufgabe 4.** Let the system of odes

$$\begin{aligned} \dot{x}_1(t) &= a_1 x_1(t), \\ \dot{x}_2(t) &= a_2 x_2(t) \end{aligned} \tag{3}$$

and let $\phi$ be the dynamical system generated by (3) i.e.,

$$\begin{aligned} &\phi : \mathbb{R} \times \mathbb{R}^2 \to \mathbb{R}^2 \\ &\phi(t, u) := \big(u_1 e^{a_1 t}, u_2 e^{a_2 t}\big), \end{aligned} \tag{4}$$

for every $t \in \mathbb{R}$ and every $u \in \mathbb{R}^2$.
Show the following.
(i) $\phi$ is $C^1$.
(ii) If $t \in \mathbb{R}$, the function $\phi_t : \mathbb{R}^2 \to \mathbb{R}^2$, defined by

$$\phi_t(u) := \phi(t, u),$$

for every $u \in \mathbb{R}^2$, is linear.
(iii) If $t = 0$, then $\phi_0 : \mathbb{R}^2 \to \mathbb{R}^2$ is the identity function on $\mathbb{R}^2$.
(iv) If $s, t \in \mathbb{R}$, then $\phi_s \circ \phi_t = \phi_{s+t}$.

**Abgabe.** Donnerstag, 17. Mai 2018 in der Vorlesung.

**Besprechung.** Montag, 28. Mai 2018, in der Übung.